Available online at www.sciencedirect.com
ScienceDirect
Robotics and Autonomous Systems 56 (2008) 583–591
Robotics and Autonomous Systems
www.elsevier.com/locate/robot

The nonholonomic redundancy of second-order nonholonomic mechanical systems

Guang-Ping He[a,b,*], Zhi-Yong Geng[a]

[a] State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, Peking University, Beijing 100871, PR China
[b] Department of Mechanical and Electrical Engineering, North China University of Technology, Beijing 100041, PR China

Received 3 November 2007; accepted 7 November 2007
Available online 17 November 2007

Abstract

The nonholonomic redundancy of second-order nonholonomic mechanical systems is investigated. It has been verified that the self-motion can be implemented demonstrably by some nonholonomic mechanical systems such as the underactuated redundant manipulators. An exponentially stabilization control method is proposed for a class of underactuated manipulators, of which the number of actuated joints is no less than that of the passive joints. It has been shown that this class of underactuated manipulators are completely controllable when the dynamic coupling of the underactuated manipulators is non-degenerated and the up-boundary of the inputs is large enough. By the proposed control method, we exhibit this class manipulators with zero weight can realize the "self-motion" as a full-actuated redundant one. As a typical application, the problem of path tracking with suppressing vibration is investigated for the underactuated redundant manipulators. It is revealed that the vibration of the underactuated redundant manipulator can be converted into an internal resonance that is compatible with the "self-motion", while it leads to no vibration at the end-effector of the manipulator. Some numerical simulations by a planar four-DOF underactuated manipulator with two actuated joints and two passive joints show the effectiveness of the accurate trajectory control method and the value of the self-motion compatible internal resonance.

Keywords: Underactuation; Manipulators; Nonholonomy; Self-motion; Elastic joints

1. Introduction

In the past, the control of nonholonomic mechanical systems has attracted much research as a challengeable problem in nonlinear control theory. The nonholonomic mechanical systems have fewer dimensions of the inputs than the dimensions of the configuration space generally. Some familiar nonholonomic systems including mobile robots [1–4], free-floating space robots [5], hopping robots in flight phase [6], spherical rolling robots [7] etc. are studied extensively. The nonholonomic constraints of these systems are first-order differential equations with Pfaffian form generally, said to be first-order nonholonomic systems. There are a plenty of research reports about motion planning and control method related to this field, such as the sinusoid method proposed by Murray and Sastry et al. [1–3], bi-directional method proposed by Nakamura and Mukherjee et al. [5], and the sliding mode control method proposed by Yang and Kim et al. [8].

Instead of the systems mentioned above, there is another class of nonholonomic systems with second-order non-integrable constraints, exampled by underactuated manipulators [9–23] or some underactuated vehicles. Oriolo and Nakamura [9] first proved that the underactuated manipulators are second-order nonholonomic mechanical system if the generalized coordinates corresponding to the passive joints were not cyclic. Arai et al. [10–13] also studied the control problem of the underactuated manipulators ten years ago. In the early years, they controlled the underactuated manipulator with the aid of brakes equipped in the passive joints [10], and extended the task to control the system in operational space for following a path [11]. For the free passive joints without brakes, they also proposed a time-scaling method [12] for solving the position

* Corresponding author at: Department of Mechanical and Electrical Engineering, North China University of Technology, Beijing 100041, PR China. Tel.: +86 010 88802835; fax: +86 010 88802835.
E-mail address: hegp55@126.com (G.-P. He).

0921-8890/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.robot.2007.11.001

control problem of an underactuated manipulator that has one passive joint based on the bi-directional motion planning technique proposed by Nakamura [5]. Almost at the same time, they also proposed a novel position control method that was composed of simple translation and rotation of the passive link with considering the motion of the center of percussion of it [13]. Similarly, De Luca and Oriolo [14] selected the center of percussion of the passive last link as the position of linearizing output, and proposed a fully linearized and input–output decoupling control method by means of a nonlinear dynamic feedback. Shiroma [15] and Lynch et al. [16] also showed the value of the center of percussion of the passive link. Based on this skill, Lynch had given a collision-free path planning method for a three-DOF manipulator with a passive last joint and realized on their experimental system. Moreover, Shiroma extended the underactuated manipulators to coupled planar rigid bodies that are hinged at the center of percussion of them, and controlled the serial rigid bodies chain by two translational acceleration inputs at the first joint. For the manipulators with one passive joint, Kobayashi and Yoshikawa [17] showed that the system is completely controllable if the first joint (in the base side) is actuated.

As a matter of fact, an underactuated manipulator cannot be restricted to one passive joint with locating at the last. One cannot expect to design an excellent system with rigorous limitations such as hinging every link at their center of percussions. De Luca and Oriolo [14] had pointed out the relationship between the center of percussion and the differential flatness. Unfortunately, the differential flatness is a special property of some underactuated mechanical systems. Murray [18] had given a catalog of nonholonomic mechanical systems with differential flatness. It can be shown that the feasible motion planning and control algorithms proposed for nonholonomic mechanical systems so far almost depend on the special properties of the systems. For instance, Xu and Ma [19] proposed a discontinuous exponentially stabilization control method for a class of underactuated system with chained form. Chung [20] and Nakamura [21] designed a chained form manipulator by spherical gear in transmission based on the control theory of chained form nonholonomic system. Scherm [22] proposed a discrete time approach for dynamic control of underactuated manipulators. This method is feasible to simple mechanical systems only because that a detailed motion planning is necessary. Suzuki [23] and Nakamura [24] suggested the averaging method can be used to construct a feasible controller for underactuated manipulator based on the nonlinear dynamics analysis and the Poincaré map. This kind of control method with periodical inputs or oscillation control methods shown by Sussmann and Liu [25–28] are elegant on theory but inconvenient in practice for the complexity of the dynamic formulations even though a simple 2R manipulator, and unfortunately, this is an approximate method such that a robust control technique is necessary.

In this paper, the exponentially stabilizable motion control method of the nonholonomic mechanical systems is investigated. We focus on the utilization of the nonholonomic redundancy of some nonholonomic mechanical systems, of which the nonholonomic redundancy can be shown by self-motion demonstrably. Nakamura [29] and his coauthors had presented that the nonholonomic redundancy is an intrinsic property of the nonholonomic mechanical system ten years ago. They simulated the optimal motion with avoiding joint limits and obstacles on a six-DOF free-floating space robot system, and exhibited the presence of nonholonomic redundancy even in the absence of ordinary kinematical redundancy. It is well known that a full-actuated redundant manipulator has kinematical redundancy and can be shown by self-motion definitely [30]. The self-motion can be used to implement some dexterous tasks such as exampled by [31–33]. For some nonholonomic redundancy systems, as stated by Nakamura, the nonholonomic redundancy is different from the ordinary kinematic redundancy and cannot be exhibited through self-motion in kinematics level. Colbaugh [34] et al. also investigated the nonholonomic redundancy of the mobile manipulator system and shown the value of nonholonomic redundancy for the motion planning and control of the system.

It is worth saying that the nonholonomic redundancy had been studied primarily for the first-order nonholonomic system with driftless. For the second-order nonholonomic mechanical systems, such as the underactuated manipulators with passive joints, which tend to be redundant in kinematics shown by many reports [12–17], but slaved by second-order nonholonomic constraints with drift term, are uncontrollable in kinematics level generally. Referring to the recent research reports [12–17], it can be concluded that the motion planning and control method of the second-order nonholonomic systems have not been solved adequately so far, the nonholonomic redundancy of the second-order nonholonomic mechanical systems has been studied less frequently.

The rest of the paper is organized as follows. In Section 2, the demonstrable self-motion of first-order nonholonomic system are shown. In Section 3, an exponentially stabilizable control method for a class of underactuated manipulator is introduced and the self-motion of this kind of second-order nonholonomic system is also displayed definitely. In Section 4, the nonholonomic redundancy is used to implement the accurate path-tracking task of the underactuated redundant manipulator with elastic active joints. The final section includes the conclusions.

2. Nonholonomic redundancy of first-order nonholonomic mechanical system

Nakamura et al. [29] pointed out that the nonholonomic redundancy could not be exhibited by self-motion as the ordinary redundancy of a full-actuated system. They conclude this based on the free-floating space robot system. We can give a simple example in conflict with this asseveration.

The rolling plate on the plane is a familiar first-order nonholonomic system, as shown in Fig. 1. Let $\boldsymbol{X} = \begin{bmatrix} x & y & \theta \end{bmatrix}^{\mathrm{T}}$ denotes the configuration of the plate, with $\begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$ being the location of the plate on the plane, θ is the angle that the plate makes with a fixed line on the plane. The constraint for the plate