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Approximate output regulation of spherical inverted pendulum by neural network control

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A B S T R A C T

The spherical inverted pendulum is a two-input, two-output non-minimum phase nonlinear system. Recently, the output regulation problem of the spherical inverted pendulum was studied in [21]. It is known that the solvability of the output regulation problem depends on the solvability of the regulator equations which are a set of nonlinear partial differential equations. Since the exact solution of the regulator equations associated with the spherical inverted pendulum is not available due to the complexity of the equations, the paper [21] tried a polynomial approximation of the solution of the regulator equations. In this paper, we first show that the solution of the regulator equations associated with the spherical inverted pendulum exist and then find an approximate solution to the output regulation problem of the spherical inverted pendulum via a neural network approximation approach. We also make some comparison between the method in this paper and the method in [21].

1. Introduction

Shown in Fig. 1 is the spherical inverted pendulum [17,21] where $x,y \in R$ represent the position of the base of the pendulum in the horizontal plane and $X,Y \in R$ represent the x and y positions of the vertical projection of the center of the pendulum onto the horizontal plane, $F_x,F_y \in R$ are the control forces being applied to the cart at the base of the pendulum, m is the mass of the uniform rod, L is the distance from the base of the pendulum to the center of mass, and g is the gravitational constant.

The motion equations of the spherical inverted pendulum are as follows [17,21]:

$$\dot{\overline{x}} = f(\overline{x}) + g(\overline{x})u$$

$$y = h(\overline{x}) \tag{1}$$

where

$$x_1 = x,\quad x_2 = \dot{x},\quad x_3 = y,\quad x_4 = \dot{y},\quad z_1 = X,\quad z_2 = \dot{X},\quad z_3 = Y,\quad z_4 = \dot{Y}$$

$$u_1 = F_x,\quad u_2 = F_y,\quad u = [u_1\ \ u_2]^T$$

$$\overline{x} = [x_1\ \ x_2\ \ x_3\ \ x_4\ \ z_1\ \ z_2\ \ z_3\ \ z_4]^T$$

$$f(\overline{x}) = \begin{bmatrix} x_2 \\ \frac{-3(L^2-z_1^2+3z_3^2)(-Bz_1+Cz_1)+12z_1z_3(-Bz_3+Cz_3)}{A} \\ x_4 \\ \frac{12z_1z_3(-Bz_1+Cz_1)-3(L^2+3z_1^2-z_3^2)(-Bz_3+Cz_3)}{A} \\ z_2 \\ \frac{3(L^2-z_1^2+3z_3^2)(-Bz_1+Cz_1)-12z_1z_3(-Bz_3+Cz_3)}{A} \\ z_4 \\ \frac{-12z_1z_3(-Bz_1+Cz_1)+3(L^2+3z_1^2-z_3^2)(-Bz_3+Cz_3)}{A} \end{bmatrix}$$

$$g(\overline{x}) = \begin{bmatrix} 0 & 0 \\ \frac{4(L^2+3z_3^2)}{A} & \frac{-12z_1z_3}{A} \\ 0 & 0 \\ \frac{-12z_1z_3}{A} & \frac{4(L^2+3z_1^2)}{A} \\ 0 & 0 \\ \frac{-3(L^2-z_1^2+3z_3^2)}{A} & \frac{12z_1z_3}{A} \\ 0 & 0 \\ \frac{12z_1z_3}{A} & \frac{-3(L^2+3z_1^2-z_3^2)}{A} \end{bmatrix}$$

$$h(\overline{x}) = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

$$A = m(L^2+3(z_1^2+z_3^2))$$

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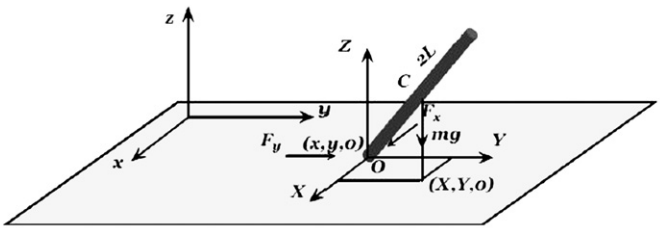

Fig. 1. Spherical inverted pendulum.

$$B=\frac{4m(L^2(z_2^2+z_4^2)-(z_2z_3-z_4z_1)^2)}{3(L^2-z_1^2-z_3^2)^2}$$

$$C=\frac{mg}{\sqrt{L^2-z_1^2-z_3^2}}$$

The stabilization problem of the spherical inverted pendulum was considered in several papers [1–3,7,16,18,22,23]. Very recently, an asymptotic tracking problem of the spherical inverted pendulum with sinusoidal reference signals was studied in [21]. The problem was formulated as a nonlinear output regulation problem as studied in, say [4,10,11,13,15,20]. It is known that the solvability of the output regulation problem depends on the solvability of the regulator equations which are a set of nonlinear partial differential equations. Typically, the exact solution of the regulator equations associated with a nonlinear system is not available. Therefore, finding various approximate solutions to the nonlinear output regulation problem has become the more practical approach. So far, there are mainly two approximate methods for the nonlinear output regulation problem. The first one is based on the polynomial approximation of the solution of the regulator equations [11,12], and the second one is based on the neural network approximation of the solution of the regulator equations [6,25]. Both methods have been tested in some single-input, single output nonlinear systems such as the ball and beam system [6,12], and the inverted pendulum on a cart system [25], pendubot system [24], and have shown quite satisfactory results. Nevertheless, as what is called "curse of dimensionality", the real challenge to any numerical approximation method is the complexity of the system.

For complex systems, either the numerical instability or the computational complexity may disqualify a method even though it may have worked very well for simple systems. From this point of view, the spherical inverted pendulum system, which has eight state variables, two inputs and two outputs and is much more complicated than most benchmark systems such as the ball and beam system and the inverted pendulum on a cart system, may serve as an interesting test-bed for the approximate approaches developed in [11,12,25]. The work in [21] has shown the performance of the polynomial approach in some detail. In this paper, we will further investigate the applicability of the neural network approach to the approximate output regulation problem of the spherical inverted pendulum, and make some comparison with the polynomial approach. Our investigation shows that the neural network approach not only offers some significant computational advantage over the polynomial approach, but also results in a much smaller approximation error for larger exogenous signals.

The rest of the paper is organized as follows. In Section 2 we summarize the basic theory of the nonlinear output regulation problem. In Section 3, we first show that the regulator equations of the system under consideration exists. Then we detail the neural network approach to obtain the approximate solution of the regulator equations of the spherical inverted pendulum. In Section 4, we design a control law that provides an approximate solution for the output regulation problem of the spherical inverted pendulum. The performance of our control law is evaluated through computer simulation and is compared with that of the control law in [21]. Finally, we close this paper with some concluding remarks.

2. Preliminaries

In this section, we summarize some results on the nonlinear output regulation problem based on the treatment in [10,13,25]. Consider a nonlinear plant as follows:

$$\dot{x}(t)=f(x(t),u(t),v(t)),\quad x(0)=x_0,\quad t\geq 0$$

$$y(t)=h(x(t),u(t),v(t)),\quad t\geq 0 \tag{2}$$

where $x(t)$ is the n-dimensional plant state, $u(t)$ the m-dimensional plant input, $y(t)$ the p-dimensional plant output representing the tracking error, $v(t)$ the q-dimensional exogenous signal representing both reference inputs and disturbances. The exogenous signal $v(t)$ is generated by a q-dimensional exosystem

$$\dot{v}(t)=a(v(t)),\quad v(0)=v_0,\quad t\geq 0 \tag{3}$$

We assume the functions f, h, and a are sufficiently smooth in a neighborhood of the origins of the respective Euclidian spaces and vanish at their origins.

We consider a state feedback control law of the following form:

$$u=\psi(x,v) \tag{4}$$

where ψ is also sufficiently smooth in a neighborhood of the origin vanishing at the origin. The composition of the plant and the control law will lead to the following closed-loop system:

$$\dot{x}(t)=f_c(x,v)\triangleq f(x,\psi(x,v),v)$$

$$y(t)=h_c(x,v)\triangleq h(x,\psi(x,v),v) \tag{5}$$

By state feedback output regulation problem, we mean the design of a controller of the form (4) such that, for all sufficiently small x_0 and v_0, the trajectories of the closed-loop system (5) exist and are bounded for all $t\geq 0$, and is such that $\lim_{t\to\infty}y(t)=0$.

Some standard assumptions for ensuring the solvability of the above problem are listed as follows.

Assumption 2.1. The pair

$$\left(\frac{\partial f}{\partial x}(0,0,0),\frac{\partial f}{\partial u}(0,0,0)\right)$$

is stabilizable.

Assumption 2.2. The equilibrium of the exosystem (3) at $v=0$ is Lyapunov stable, and all the eigenvalues of $(\partial a/\partial v)(0)$ have zero real parts.

Assumption 2.3. There exist two sufficiently smooth functions $\mathbf{x}(v)$ and $\mathbf{u}(v)$ defined in a neighborhood V of the origin of R^q such that $\mathbf{x}(0)=0,\mathbf{u}(0)=0$ and for all $v\in V$

$$\frac{\partial\mathbf{x}(v)}{\partial v}a(v)=f(\mathbf{x}(v),\mathbf{u}(v),v)$$

$$0=h(\mathbf{x}(v),\mathbf{u}(v),v) \tag{6}$$

Remark 2.1. Eq. (6) is known as regulator equations [13]. Assumptions 2.3 is a necessary condition for the solvability of the nonlinear output regulation problem. Moreover, Assumption 2.3 together with Assumptions 2.1 and 2.2 also leads to a state feedback control law of the form

$$\psi(x,v)=\mathbf{u}(v)+K(x-\mathbf{x}(v)) \tag{7}$$