Neurocomputing 85 (2012) 53–61

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

Exponential synchronization for delayed chaotic neural networks with nonlinear hybrid coupling ☆

Guobao Zhang [a,*], Ting Wang [a], Tao Li [b], Shumin Fei [a]

[a] Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing 210096, China
[b] School of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

ARTICLE INFO

Article history:
Received 5 September 2011
Received in revised form
7 November 2011
Accepted 28 December 2011
Communicated by N. Ozcan
Available online 18 January 2012

Keywords:
Delayed neural networks
Lyapunov–Krasovskii functional (LKF)
Exponential synchronization
Reciprocal convex technique
LMI approach

ABSTRACT

This paper deals with global exponential synchronization in arrays of coupled delayed chaotic neural networks with nonlinear hybrid coupling. Through constructing one novel Lyapunov–Krasovskii functional, two novel synchronization criteria are presented in terms of linear matrix inequalities (LMIs) based on reciprocal convex technique, and these conditions are heavily dependent on the bounds of both time-delay and its derivative. Through employing LMI in Matlab Toolbox and adjusting some matrix parameters in the derived results, the design and applications of the generalized networks can be realized, which shows that our methods can improve some reported methods. The efficiency and applicability of the proposed methods can be demonstrated by three numerical examples with simulations.

1. Introduction

In past decades, synchronization of various chaotic systems has gained considerable attention since the pioneering works of Pecora and Carroll appeared [1,2]. Presently, it is widely known that many benefits of having synchronization or chaos synchronization in various engineering fields can be existent such as secure communication, image processing, harmonic oscillation generation, and so on. Also, the existence of synchronization in language emergence and development results can help come up with the common vocabulary and agents' synchronization in organization management can improve their work efficiency. In recent years, the problem on synchronization in chaotic systems has been extensively investigated owing to the potential applications in various engineering areas [6–34]. Especially, since chaos synchronization in arrays of linearly coupled dynamical systems was firstly considered by [3], arrays of coupled systems including coupled delayed chaotic ones have attracted the researchers' attention as they can exhibit some interesting phenomena [4,5], and many elegant results have been derived in the present literature, see [8–34] and the references therein.

As a typical complex networks, delayed neural networks (DNNs) have been verified to exhibit some complex and unpredictable behaviors such as stable equilibria, periodic oscillations, bifurcation, and chaotic attractors. Thus chaos synchronization for arrays of coupled DNNs have been discussed widely and many elegant results have been proposed in [9–34] recently. In [9], by applying adaptive feedback controllers, the paper has studied the global synchronization of coupled complex networks with delayed coupling based on pinning control. The stability of synchronized state has been studied in arbitrarily coupled delayed complex networks in [10], where coupling configurations are not assumed to be symmetric and irreducible. The synchronization of linearly coupled DNNs was investigated in [11], in which the dynamical behavior of the uncoupled system can be chaotic or others and the coupling configuration is variable. The authors in [12] have considered the robust synchronization of coupled DNNs under general impulsive control. In [13], this paper has proposed an adaptive procedure to the synchronization for coupled identical Yang–Yang type fuzzy DNNs based on one simple adaptive controller. In [14], with all parameters unknown, the authors studied the robust synchronization between two coupled DNNs that were linearly and unidirectionally coupled. Owing to those above-mentioned results were presented via some complicated inequalities, which makes them uneasily checked and applied to real cases.

Later, through combining LMI approach and Kronecker product, as for various coupled DNNs, some easy-to-check criteria were established for some kinds of synchronization such as asymptotical

☆ This work is supported by the National Natural Science Foundation of China under Grants 60905009, 61004032, 61172135, the Talent Introduction Foundation of Nanjing University of Aeronautics and Astronautics under Grant 1003-56YAH11055, and the Special Foundation of China Postdoctoral Science Foundation Projects under Grant 201003546.

* Corresponding author.
E-mail addresses: zhangguobao@yahoo.cn (G. Zhang), chunchun1010@163.com (T. Wang), huainanlitao@yahoo.com.cn (T. Li), smfei@seu.edu.cn (S. Fei).

0925-2312/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.neucom.2011.12.028

synchronization, cluster synchronization, robust synchronization, and exponential one [15–34]. Yet, the system forms in [15–26] seemed simple and the inner linking matrices needed to be symmetric, which make those derived results unapplicable to tackle more general DNNs. Though the authors have studied the exponential synchronization for coupled DNNs of general forms and the convex technique was utilized in [27], its condition on behaved function of addressed networks was very strict and the inner linking matrices were symmetric. Based on LMI approach, the authors have studied the exponential synchronization for delayed neural networks in [28–30] and some elegant results have been given, in which unbounded distributed delay and impulsive effects were involved. On the other hand, since hybrid coupling was firstly introduced in [31], some good synchronization results have been given to improve its discussion for coupled DNNs of simple forms in [32–34]. Presently, as for time variable delay, reciprocal convex approach has been proven to be very efficient in reducing the conservatism [35]. To our best knowledge, with both available bounds of time delay and its derivative, few authors have used the combination of the reciprocal convex technique and convex one to deal with the global synchronization for coupled chaotic DNNs and the inner linking matrices unnecessarily symmetric, which constitutes the main focus of this work.

In this paper, the global exponential synchronization of N identical chaotic neural networks with nonlinear hybrid coupling is studied and two novel conditions are derived by utilizing LMI form. It shows that the chaos synchronization of coupled networks is ensured by a suitable design of inner coupled linking matrix and inner delayed coupled linking ones. The addressed systems can include some famous network models as its special cases and the combined convex technique are employed to reduce the conservatism. Finally, the efficiency of the synchronization criteria can be demonstrated by utilizing three numerical examples.

Notations: $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For the symmetric matrices $X,Y,X>Y$(respectively, $X\geq Y$) means that $X-Y>0(X-Y\geq 0)$ is a positive-definite (respectively, positive-semidefinite) matrix; A^T stands for the transpose of matrix A; I_m represents the $m\times m$ identity matrix; and $\begin{bmatrix} X & Y \\ Y^T & Z\end{bmatrix}=\begin{bmatrix} X & Y \\ * & Z\end{bmatrix}$ with $*$ denoting the symmetric term in a symmetric matrix.

2. Problem formulations

Suppose the nodes are coupled with states $x_i(t),i\in\{1,\dots,N\}$, we consider the following dynamical networks with each node being an n-dimensional delayed chaotic neural networks described by

$$\begin{aligned}\dot{x}_i(t)&=-\overline{\beta}(x_i(t))+A\overline{f}(x_i(t))+B\overline{f}(x_i(t-\tau(t)))+\mathrm{I}(t)\\&\quad+\sum_{j=1}^{N}l_{ij}^1G\overline{h}(x_j(t))+\sum_{j=1}^{N}l_{ij}^2H\overline{h}(x_j(t-\tau(t)))\\&\quad+\sum_{j=1}^{N}l_{ij}^3K\int_{t-\tau(t)}^{t}\overline{h}(x_j(s))\,ds,\end{aligned}\tag{1}$$

where $x_i(t)=[x_{i1}(t),x_{i2}(t),\dots,x_{in}(t)]^T$ are the state vectors, here $A=[a_{ij}]_{n\times n},B=[b_{ij}]_{n\times n}$, $\overline{\beta}(x_i)=[\overline{\beta}_1(x_{i1}),\dots,\overline{\beta}_n(x_{in})]^T$ stand for the behaved functions, $\overline{f}(x_i(\cdot))=[\overline{f}_1(x_{i1}(\cdot)),\dots\overline{f}_n(x_{in}(\cdot))]^T$ are the activation functions, $\mathrm{I}(t)=[\mathrm{I}_1(t),\dots,\mathrm{I}_n(t)]^T\in\mathbf{R}^n$ is the external input vector, and $\overline{h}(x_i(\cdot))=[\overline{h}_1(x_{i1}(\cdot)),\dots,\overline{h}_n(x_{in}(\cdot))]^T$ are the nonlinear functions; here we also assume $G=\mathrm{diag}(g_1,\dots,g_n)\geq 0,H=\mathrm{diag}(h_1,\dots,h_n)\geq 0$, and $K=\mathrm{diag}(k_1,\dots,k_n)\geq 0$ to represent the inner coupling matrices.

For the dynamical networks (1), the following assumptions are utilized throughout this paper.

Assumption 1. Here $\tau(t)$ denotes the interval time-varying delay satisfying

$$0\leq\tau_0\leq\tau(t)\leq\tau_m,\quad \mu_0\leq\dot{\tau}(t)\leq\mu_m<+\infty,\tag{2}$$

and we set $\overline{\tau}_m=\tau_m-\tau_0$ and $\overline{\mu}_m=\mu_m-\mu_0$.

Assumption 2. $L^k=[l_{ij}^k]_{N\times N}$ $(k=1,2,3)$ is the configuration matrix that is irreducible and satisfies

$$l_{ij}^k\geq 0,\quad l_{ii}^k=-\sum_{j=1,j\neq i}^{N}l_{ij}^k,\ i\neq j;\ i,j=1,\dots,N;\ k=1,2,3,\tag{3}$$

here $l_{ij}^k>0$, if there exists a connection between node i and the one j and otherwise, $l_{ij}^k=0$.

Assumption 3. Each function $\overline{\beta}_i(\cdot):\mathbf{R}\to\mathbf{R}$ is globally Lipschitz and there exists γ_i such that $0\leq\gamma_i\leq\dot{\overline{\beta}}_i(z)<+\infty$ for all $z\in\mathbf{R}$. Here, we denote $\Gamma=\mathrm{diag}(\gamma_1,\dots,\gamma_n)$.

Assumption 4. For any $\alpha,\beta\in\mathbf{R}$, the nonlinear functions $\overline{f}_i(\cdot)$ and $\overline{h}_i(\cdot)$ satisfy the following conditions:

$$[\overline{f}_i(\alpha)-\overline{f}_i(\beta)-\sigma_i^+(\alpha-\beta)][\overline{f}_i(\alpha)-\overline{f}_i(\beta)-\sigma_i^-(\alpha-\beta)]\leq 0,$$

$$[\overline{h}_i(\alpha)-\overline{h}_i(\beta)-\delta_i^+(\alpha-\beta)][\overline{h}_i(\alpha)-\overline{h}_i(\beta)-\delta_i^-(\alpha-\beta)]\leq 0,\quad i=1,\dots,n,$$

where $\sigma_i^+,\sigma_i^-,\delta_i^+,\delta_i^-$ are given constants. Here we introduce the following denotations $\overline{\Sigma}=\mathrm{diag}(\sigma_1^+,\dots,\sigma_n^+)$, $\Sigma=\mathrm{diag}(\sigma_1^-,\dots,\sigma_n^-)$, $\overline{\delta}_i=(\delta_i^++\delta_i^-)/2$, and

$$\Sigma_1=\mathrm{diag}(\sigma_1^+\sigma_1^-,\dots,\sigma_n^+\sigma_n^-),\quad \Sigma_2=\mathrm{diag}\left(\frac{\sigma_1^++\sigma_1^-}{2},\dots,\frac{\sigma_n^++\sigma_n^-}{2}\right).$$

Suppose that the complex networks (1) will approach the desired inhomogeneous state defined by $x_1(t),\dots,x_N(t)\to s(t)$, i.e., $s(t)\in\mathrm{R}^n$ is the desired synchronization state. The function $s(t)$ is defined as

$$\dot{s}(t)=-\overline{\beta}(s(t))+A\overline{f}(s(t))+B\overline{f}(s(t-\tau(t)))+\mathrm{I}(t).\tag{4}$$

Let $\varepsilon_i(t)=x_i(t)-s(t)$, $\beta(\varepsilon_i(t))=\overline{\beta}(\varepsilon_i(t)+s(t))-\overline{\beta}(s(t)),f(\varepsilon_i(\cdot))=\overline{f}(\varepsilon_i(\cdot)+s(\cdot))-\overline{f}(s(\cdot))$, and $h(\varepsilon_i(\cdot))=\overline{h}(\varepsilon_i(\cdot)+s(\cdot))-\overline{h}(s(\cdot))$, then one can easily check that

$$\begin{aligned}\sum_{j=1}^{N}l_{ij}^1G\overline{h}(x_j(t))&=\sum_{j=1}^{N}l_{ij}^1G\overline{h}(\varepsilon_j(t)+s(\cdot))\\&=\sum_{j=1}^{N}l_{ij}^1Gh(\varepsilon_j(t))+\sum_{j=1}^{N}l_{ij}^1G\overline{h}(s(t))=\sum_{j=1}^{N}l_{ij}^1Gh(\varepsilon_j(t)).\end{aligned}\tag{5}$$

Thus it follows from (5) that

$$\begin{aligned}&\sum_{j=1}^{N}l_{ij}^2H\overline{h}(x_j(t-\tau(t)))=\sum_{j=1}^{N}l_{ij}^2Hh(\varepsilon_j(t-\tau(t))),\\&\sum_{j=1}^{N}l_{ij}^3K\int_{t-\tau(t)}^{t}\overline{h}(x_j(s))\,ds=\sum_{j=1}^{N}l_{ij}^3K\int_{t-\tau(t)}^{t}h(\varepsilon_j(s))\,ds.\end{aligned}\tag{6}$$

Subtracting (4) from (1) and using (5)–(6), we can derive

$$\begin{aligned}\dot{\varepsilon}_i(t)&=-\beta(\varepsilon_i(t))+Af(\varepsilon_i(t))+Bf(\varepsilon_i(t-\tau(t)))+\sum_{j=1}^{N}l_{ij}^1Gh(\varepsilon_j(t))\\&\quad+\sum_{j=1}^{N}l_{ij}^2Hh(\varepsilon_j(t-\tau(t)))+\sum_{j=1}^{N}l_{ij}^3K\int_{t-\tau(t)}^{t}h(\varepsilon_j(s))\,ds,\ i=1,2,3.\end{aligned}\tag{7}$$

It is easy to verify that the functions $\beta(\varepsilon_i(t))$ and $f(\varepsilon_i(\cdot))$ satisfy Assumptions 3 and 4. In what follows, some useful basic definitions and denotation need to be introduced.