Neurocomputing 73 (2010) 3139–3146

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Anti-control of Hopf bifurcation for Chen's system through washout filters ☆

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ARTICLE INFO

Article history:
Received 5 November 2009
Received in revised form
12 March 2010
Accepted 19 June 2010
Communicated by H. Jiang
Available online 27 July 2010

Keywords:
Hopf bifurcation
Periodic solution
Chen's system
Anti-controlling bifurcations

ABSTRACT

In this paper, we consider the problem of anti-controlling bifurcations for Chen's system, that is, a certain bifurcation is created at a desired location with preferred properties by appropriate control. Washout-filter-aided dynamic feedback control laws are developed for the creation of Hopf bifurcations. By choosing appropriate control parameter, we investigate the existence of Hopf bifurcation. Conditions are given to determine the directions and stabilities of the bifurcating periodic solutions. We also find that this control laws can be applied to Chen's system for the purpose of control and anti-control of chaotic attractor. Finally, some numerical simulations are given to illustrate the effectiveness of the results found.

1. Introduction

Bifurcation control refers to the task of designing a controller to suppress or reduce some existing bifurcation dynamics of a given nonlinear system, thereby achieving some desirable dynamical behaviors [3]. Anti-control of bifurcation, as opposed to the direct control, means a certain bifurcation is created at a desired location with preferred properties by appropriate control. Our work on anti-control of bifurcations is motivated by observations that in some applications, it may be advantageous to introduce new bifurcations to the nominal branch of system output [1,2]. It is now known that bifurcation properties of a system can be modified via various feedback control methods. One of the representative approaches is applied a washout filter-aided dynamic feedback controller. The use of washout filters ensures that low frequency orbits of the system are retained in the closed loop system, with only the transient dynamics and higher frequency orbits modified. Benefit of using washout filters is that all the equilibrium points of the open-loop system are preserved. Washout filters have been used in feedback control in many applications [9,15,20,21].

Over the last three years, there are extensive studies on Chen's system (1) (see, for example [4,6,7,10–13,17,19]), controlling chaos [14] and bifurcation theory [5,18]. In the present paper, we are interesting in control of Chen's chaotic attractor. By applying the washout filter-based approach and constructing a controlled system with delay, we investigate the effect of delay on the dynamical behavior of Chen's system. By employing normal form theory and center manifold theorem, the stability of bifurcating periodic solutions for this controlled system are studied in detail.

The remainder of this paper is organized as follows. In Section 2, a model of controlled Chen's system through washout filter is created. The stability and the existence of Hopf bifurcation parameter are determined in Section 3. In Section 4, based on the normal form method and the center manifold theorem introduced by Hassard et al. [8], the direction, stability and the period of the bifurcating periodic solutions are analyzed. To verify the theoretic analysis, numerical simulations are given in Section 5. Finally, Section 6 concludes with some discussions.

2. Control model

The original Chen's system is described by the following three-dimensional smooth quadratic autonomous system [4]:

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = (c-a)x - xz + cy, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

which is chaotic when $a=35$, $b=3$, $c=28$.

☆ This work was jointly supported by the National Natural Science Foundation of China under Grant 60874088, the Natural Science Foundation of Jiangsu Province of China under Grant BK2009271, and the project of Shandong province higher educational science and technology program, no. J09LA56.

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doi:10.1016/j.neucom.2010.06.016

We consider a general form of dynamical system

$$\dot{X} = f(X;\mu), \tag{2}$$

where X is a vector, μ is a parameter. The washout-filter-aided controller assumes the following structure [2]:

$$\begin{cases} \dot{X} = f(\mathbf{X};\mu)+u, \\ \dot{w} = \mathbf{X}_i - dw \triangleq \rho, \\ u = g(\rho;\mathbf{K}), \end{cases} \tag{3}$$

where u is a control input, g is a control function and d is the washout filter time constant. The following constraints should be fulfilled: $d>0$, which guarantees the stability of the washout filter; $g(0,\mathbf{K})=0$, which preserves the original equilibrium points.

In this paper, the controlled system is designed as follows:

$$\begin{cases} \dot{x}(t) = a(y(t)-x(t)), \\ \dot{y}(t) = (c-a)x(t)-x(t)z(t)+cy(t)+u(t), \\ \dot{z}(t) = x(t)y(t)-bz(t), \\ \dot{u}(t) = \alpha[y(t)-y(t-\tau)]-du. \end{cases} \tag{4}$$

This system has the similar character with washout filter control:

- $d>0$, which guarantees the stability of the controller.
- The original equilibrium points were preserved.

Our next approach is to creation Hopf bifurcations using the given feedback control laws.

3. Existence of Hopf bifurcation

In this section, we choose the gain α as a constant and investigate the effect of time-delay τ on the dynamic behavior of the controlled system (4). The following conclusions for the uncontrolled Chen's system are needed:

Lemma 1 (*Song and Wei [16]*). *Suppose that $a>0, b>0, c>0$,*

(1) *If $a>2c$, then system (1) has one steady state $S_0(0,0,0)$ and it is asymptotically stable.*
(2) *If $a<2c$, then system (1) has three real steady states: $S_-(-x_0,-y_0,z_0)$, $S_0(0,0,0)$ and $S_+(x_0,y_0,z_0)$. The steady state S_+ (or S_-) of system (1) is stable when $b>(c^2+3ac-2a^2)/c$, and the corresponding characteristic equation has three eigenvalues: one negative real root and one pair of conjugate complex roots with negative real parts.*

3.1. Hopf bifurcation from the steady state S_0

The linear equation of the controlled system (4) at S_0 (where $u_0=0$) is

$$\begin{cases} \dot{x}(t) = a(y(t)-x(t)), \\ \dot{y}(t) = (c-a)x(t)+cy(t)+u(t), \\ \dot{z}(t) = -bz(t), \\ \dot{u}(t) = \alpha[y(t)-y(t-\tau)]-du. \end{cases} \tag{5}$$

The associated characteristic equation of the linearized system is

$$\det\begin{pmatrix} \lambda+a & -a & 0 & 0 \\ a-c & \lambda-c & 0 & -1 \\ 0 & 0 & \lambda+b & 0 \\ 0 & \alpha(e^{-\lambda\tau}-1) & 0 & \lambda+d \end{pmatrix} = 0. \tag{6}$$

That is

$$(\lambda+b)\{\lambda^3+(a-c+d)\lambda^2+(a(a-2c)+(a-c)d-\alpha)\lambda + a[(a-c)d-\alpha]+\alpha(\lambda+a)e^{-\lambda\tau}\} = 0. \tag{7}$$

It is well known that the equilibrium (0,0,0,0) is stable if all the roots of (7) have negative real parts. Obviously, Eq. (7) always has a negative root $\lambda=-b$ for all $\tau\geq 0$, so we only need to investigate the third degree transcendental polynomial equation

$$\lambda^3+(a-c+d)\lambda^2+(a(a-2c)+(a-c)d-\alpha)\lambda + a[(a-c)d-\alpha]+\alpha(\lambda+a)e^{-\lambda\tau} = 0 \tag{8}$$

or

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3+(b_0\lambda+b_1)e^{-\lambda\tau} = 0, \tag{9}$$

where

$$a_1 = a-c+d, \quad a_2 = a(a-2c)+(a-c)d-\alpha, \quad a_3 = a[(a-c)d-\alpha], \quad b_0=\alpha, \quad b_1 = a\alpha.$$

We first examine when the characteristic equation (7) has pairs of pure imaginary roots. If $\lambda=\pm i\omega$ with $\omega>0$, then we have

$$-i\omega^3+a_1\omega^2+ia_2\omega+a_3+(ib_0\omega+b_1)(\cos\omega\tau - i\sin\omega\tau) = 0.$$

Separating the real and imaginary parts, we have

$$\begin{cases} a_1\omega^2-a_3 = b_1\cos\omega\tau+b_0\omega\sin\omega\tau, \\ \omega^3-a_2\omega = -b_1\sin\omega\tau+b_0\omega\cos\omega\tau, \end{cases} \tag{10}$$

which lead to

$$\omega^6+(a_1^2-2a_2)\omega^4+(a_2^2-b_0^2-2a_1a_3)\omega^2+a_3^2-b_1^2 = 0. \tag{11}$$

Let $z=\omega^2$ and denote

$$p = a_1^2-2a_2, \quad q = a_2^2-b_0^2-2a_1a_3, \quad r = a_3^2-b_1^2.$$

Then, Eq. (11) becomes

$$z^3+pz^2+qz+r = 0. \tag{12}$$

In the following, we need to seek conditions such that Eq. (12) has at least one positive root. Denote

$$h(z) = z^3+pz^2+qz+r.$$

Lemma 2 (*Song and Wei [16]*). *For Eq. (12), we have the following results.*

(1) *if $r<0$, then Eq. (12) has at least one positive root.*
(2) *if $r\geq 0$ and $\Delta = p^2-3q\leq 0$, then Eq. (12) has no positive root.*
(3) *if $r\geq 0$ and $\Delta = p^2-3q>0$, then Eq. (12) has positive roots if and only if $z_1^* = \frac{1}{3}(-p+\sqrt{\Delta})>0$ and $h(z_1^*)\leq 0$.*

Suppose that z_k is a positive root of Eq. (12), then $\omega_k = \pm\sqrt{z_k}$. From Eq. (10), we have

$$\tau_k^{(j)} = \frac{1}{\omega_k}\left\{\arccos\frac{b_0\omega_k^4+(b_1a_1-b_0a_2)\omega_k^2-b_1a_3}{b_1^2+b_0^2\omega_k^2}+2\pi j\right\}. \tag{13}$$

In order to create a Hopf bifurcation from the bifurcation point, the following *transversality condition* is needed:

$$\left.\frac{d(\mathrm{Re}\lambda)}{d\tau}\right|_{\tau=\tau_k^{(j)}} \neq 0. \tag{14}$$

Substituting $\lambda(\tau)$ into Eq. (9) and taking the derivative with respect to τ, we obtain

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{3\lambda^2+2a_1\lambda+a_2}{\lambda(b_0\lambda+b_1)}e^{\lambda\tau}+\frac{b_0}{\lambda(b_0\lambda+b_1)}-\frac{\tau}{\lambda}. \tag{15}$$