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Dynamics of solution for a class of delayed diffusive neural networks with mixed boundary conditions ☆

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ABSTRACT

In this paper, we investigate a class of cellular neural networks model with delays and diffusive terms. By using the method of upper and lower solutions, we obtain that if the neuronal output signal functions in system possess mixed quasimonotone property and the corresponding elliptic system has upper and lower solutions the model has a unique nonconstant equilibrium solution. Under some additional conditions we further obtain that the solution of the neural networks converges to this nonconstant equilibrium solution.

1. Introduction

In recent years, many authors have been investigating the dynamical behaviors of cellular neural networks with or without diffusive terms due to their applications in associative memory, parallel computation, pattern recognitions, signal processing and optimization problems [1,2]. In fact, earlier applications of neural networks to optimization problems have suffered from the existence of a set of equilibria [3,4]. And, the global attract of unique equilibrium for neural networks is also of great importance [5,6]. A qualitative analysis of the global asymptotically stable equilibrium point for cellular neural networks is of wide interests and has been the concern of many authors [7–10]. Studies on cellular neural networks not only involve a discussion of stabilities, but also involve many dynamic behavior such as periodic oscillatory behavior and bifurcation [11]. Furthermore, equilibrium and asymptotic behavior of different class of neural networks such as Cohen–Grossberg neural networks, Hopfield neural networks and recurrent neural networks have been intensively studied in the past decades [12–14].

However, strictly speaking, diffusion effect cannot be avoided in neural networks when electrons are moving in asymmetric electromagnetic field. In [15–21], the stability of neural networks with diffusive terms, which are expressed by partial differential equations, has been considered. And the most of boundary conditions of investigated diffusive neural networks [15–22] are the Neumann boundary conditions. To our knowledge, there are few studies on diffusive neural networks with Dirichlet boundary conditions [23,24].

Furthermore, most of the discussions in the former works are the existence and uniqueness of constant equilibrium point and its properties of asymptotic behavior. In [23,24], the author discussed global exponential stability of constant equilibrium point for the following reaction-diffusion delayed neural networks with Dirichlet boundary conditions:

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} = \displaystyle\sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(D_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right) - c_i u_i(t,x) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t,x)) \\ \qquad + \displaystyle\sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_j,x)) + I_i & (t>0, x\in\Omega), \\ u_i = 0 & (t>0, x\in\partial\Omega), \\ u_i(t,x) = \eta_i(t,x) \geq 0 & (-\tau_i \leq t \leq 0, x\in\Omega) \\ & (i=1,\ldots,n). \end{cases}$$

We find that constant equilibrium point of the above system is only equal to $(0,0,\ldots,0)^T$ since it is restricted by Dirichlet boundary conditions. Thus, from the equations we know that the equations $\sum_{j=1}^{n} a_{ij} f_j(0) + \sum_{j=1}^{n} b_{ij} g_j(0) + I_i = 0$ must hold in order to ensure the existence of constant equilibrium point. Generally, this condition is difficult to be satisfied in reality.

Motivated by the above discussion, in this paper, we consider the following delayed diffusive neural networks with mixed

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boundary conditions:

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} = \sum_{l=1}^{m} \dfrac{\partial}{\partial x_l}\left(D_{il}\dfrac{\partial u_i(t,x)}{\partial x_l}\right) - c_i u_i(t,x) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t,x)) \\ \qquad + \sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_j,x)) + I_i & (t>0, x\in\Omega), \\ B_i u_i = 0 & (t>0, x\in\partial\Omega), \\ u_i(t,x) = \eta_i(t,x) \ge 0 & (-\tau_i \le t \le 0, x\in\Omega) \\ & (i=1,\ldots,n). \end{cases} \tag{1}$$

The operators B_i are given by

$$B_i u_i = \alpha_i \frac{\partial u_i}{\partial n} + \beta_i(x) u_i \quad (i=1,\ldots,n),$$

where for each i (i=1,2,…,n), B_i is of either Dirichlet type ($\alpha_i = 0, \beta_i(x) = 1$), or Neumann–Robin type ($\alpha_i = 1, \beta_i(x) \ge 0$).

We firstly discuss the nonconstant equilibrium solution of system (1) which is solution of corresponding elliptic system in relation to system (1)

$$\begin{cases} -\sum_{l=1}^{m} \dfrac{\partial}{\partial x_l}\left(D_{il}\dfrac{\partial u_i(x)}{\partial x_l}\right) = -c_i u_i(x) + \sum_{j=1}^{n} a_{ij} f_j(u_j(x)) \\ \quad + \sum_{j=1}^{n} b_{ij} g_j(u_j(x)) + I_i & (x\in\Omega), \\ B_i u_i = 0 & (x\in\partial\Omega) \quad (i=1,\ldots,n). \end{cases} \tag{2}$$

We discuss that for given initial conditions, system (1) exist unique solution and it is convergent to its corresponding nonconstant equilibrium solution as $t\to\infty$ under some conditions. The previous investigations about diffusive neural networks are either Dirichlet boundary conditions or Neumann conditions. That is to say, we consider here the more general boundary conditions which conclude in the above boundary conditions.

In the past and recent years, the main methods investigating asymptotic behavior of neural networks are Lyapunov functional method, inequalities technique and degree theory. In this paper, we use the method of upper and lower solutions to obtain asymptotic behavior of solution of system (1). These results are independent of the time delays.

The organization of this paper is as follows. Firstly, we introduce some necessary notions and preliminaries in Section 2. In Section 3, by using the method of upper and lower solutions, asymptotic behavior of solution of system (1) are given. An example is presented in Section 4 to demonstrate the main results.

2. Model description and preliminaries

Let $\overline{\Omega}$ be the closure of Ω, and for any finite $T>0$ and i=1,…,n, we set

$$D_T = (0,T]\times\Omega, \quad \overline{D}_T = [0,T]\times\overline{\Omega}, \quad S_T = (0,T]\times\partial\Omega,$$

$$Q_0^{(i)} = [-\tau_i, 0]\times\Omega, \quad Q_0 = Q_0^{(1)}\times\cdots\times Q_0^{(n)}.$$

In system (1), Ω is a bounded domain in R^n with boundary $\partial\Omega$. $u_i(t,x)$ is the state of the i-th neuron at time t and in space x. $c_i>0$ represents the rate with which the i-th unit will reset its potential to the resting state in isolation when disconnected from the networks and external inputs, I_i represents the external bias on the i-th unit, f_j and g_j denotes the output of the j-th unit, τ_j is the transmission delay of the j-th unit from the i-th unit at time t, a_{ij} is the strength of the neuron interconnection within the networks and b_{ij} is the interconnection with delay τ_j. Smooth function $D_{il}(x)>0$ represents the transmission diffusion operator along the i-th neuron, Δ is Laplacian operator, and here we suppose a no-flux (Neumann) boundary condition and nonnegative initial distribution.

Throughout the paper, we assume that for each l=1,…,m, $D_{il}(x)$ and its first partial derivatives are in $C^{\alpha}(\overline{\Omega})$, $\alpha\in(0,1)$. The boundary coefficient β_i is in $C^{1+\alpha}(\partial\Omega)$, and $\partial\Omega$ is of class $C^{1+\alpha}$. We also assume that η_i is Holder continuous on $\partial\Omega$ and Q_0^i, and satisfies the compatibility condition at t=0 when $\alpha_i=0$.

First of all, we give some definitions for a system of parabolic equations with discrete time delays which is given in the form

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} - L_i u_i = F_i(x,u,u_\tau) & \text{in } D_T, \\ B_i u_i = h_i(x) & \text{on } S_T, \\ u_i(t,x) = \eta_i(t,x) \ge 0 & \text{in } Q_0^{(i)} \quad (i=1,\ldots,n), \end{cases} \tag{3}$$

where u=(($u_1(t,x)$,…,$u_n(t,x)$)), $u_\tau = ((u_1(t-\tau,x),\ldots,u_n(t-\tau,x))$. The operator L_i is given by

$$L_i = \sum_{j,k=1}^{n} a_{jk}^{(i)}(x)\frac{\partial^2}{\partial x_j\,\partial x_k} + \sum_{j=1}^{n} b_j^{(i)}(x)\frac{\partial}{\partial x_j} \quad (i=1,\ldots,n).$$

It is assumed that for each i=1,…,n, L_i is a uniformly elliptic operator in $\overline{\Omega}$. The corresponding elliptic system in relation to system (3) is of the form

$$\begin{cases} -L_i u_i = F_i(x,u,u) & (x\in\Omega), \\ B_i u_i = h_i(x) & (x\in\partial\Omega) \quad (i=1,\ldots,n). \end{cases} \tag{4}$$

Assuming that $F(x,u,v)$=($F_1(x,u,v)$,…,$F_n(x,u,v)$) is Hölder continuous in x and continuously differentiable in u and v for u, v in some bounded subset Ω of R^n where u=($u_1(t,x)$,…,$u_n(t,x)$) and v=($v_1(t,x)$,…,$v_n(t,x)$). Specially, by writing u and v in split forms

$$u = (u_i, [u]_{a_i}, [u]_{b_i}), \quad v = ([v]_{c_i}, [v]_{d_i}),$$

where a_i, b_i, c_i and d_i are some nonnegative integers with $a_i+b_i=n-1$ and $c_i+di=n$. Hence, we can rewrite systems (3) and (4) in the form

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} - L_i u_i = F_i(x, u_i, [u]_{a_i}, [u]_{b_i}, [u_\tau]_{c_i}, [u_\tau]_{d_i}) & \text{in } D_T, \\ B_i u_i = h_i(x) & \text{on } S_T, \\ u_i(t,x) = \eta_i(t,x) \ge 0 & \text{in } Q_0^{(i)} \quad (i=1,\ldots,n) \end{cases} \tag{5}$$

and

$$\begin{cases} -L_i u_i = F_i(x, u_i, [u]_{a_i}, [u]_{b_i}, [u]_{c_i}, [u]_{d_i}) & (x\in\Omega), \\ B_i u_i = h_i(x) & (x\in\partial\Omega) \quad (i=1,\ldots,n). \end{cases} \tag{6}$$

Definition 1. A vector function $F(x,u,v)$ is said to be mixed quasimonotone in some subset Λ of R^n if for each i=1,…,n, there exist nonnegative integers a_i, b_i, c_i and d_i with $a_i+b_i=n-1$ and $c_i+d_i=n$ such that for every $u=(u_i,[u]_{a_i},[u]_{b_i})$ and $v=([v]_{c_i},[v]_{d_i})$ in Λ, $F_i(x,u,v)$ is monotone nondecreasing in $[u]_{a_i}$ and $[v]_{c_i}$ and monotone nonincreasing in $[u]_{b_i}$ and $[v]_{d_i}$. The function $F(x,u,v)$ is said to be quasimonotone nondecreasing in Λ if $b_i=d_i=0$ for all i.

Definition 2. A pair of $\tilde{u}=(\tilde{u}_1,\ldots,\tilde{u}_n)$, $\hat{u}=(\hat{u}_1,\ldots,\hat{u}_n)$ is called coupled upper and lower solutions of (5) if $\tilde{u}\ge\hat{u}$ and if

$$\begin{cases} \dfrac{\partial \tilde{u}_i(t,x)}{\partial t} - L_i\tilde{u}_i \ge F_i(x,\tilde{u}_i,[\tilde{u}]_{a_i},[\hat{u}]_{b_i},[\tilde{u}_\tau]_{c_i},[\hat{u}_\tau]_{d_i}) \\ \dfrac{\partial \hat{u}_i(t,x)}{\partial t} - L_i\hat{u}_i \le F_i(x,\hat{u}_i,[\hat{u}]_{a_i},[\tilde{u}]_{b_i},[\hat{u}_\tau]_{c_i},[\tilde{u}_\tau]_{d_i}) & \text{in } D_T, \\ B_i\hat{u}_i \le h_i(x) \le B_i\tilde{u}_i & \text{on } S_T, \\ \hat{u}_i(t,x) \le \eta_i(t,x) \le \tilde{u}_i(t,x) & \text{in } Q_0^{(i)} \quad (i=1,\ldots,n). \end{cases} \tag{7}$$