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Letters

Effects of "rich-gets-richer" rule on small-world networks

Hongwei Dai [a,*], Shangce Gao [b], Yu Yang [a], Zheng Tang [b]

[a] School of Computer Engineering, Huaihai Institute of Technology, Lianyungang 222005, China
[b] Faculty of Engineering, University of Toyama, Gofuku 3190, Toyama 930-8555, Japan

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ABSTRACT

In this paper, we propose a new rewiring rule that generates small-world networks with larger clustering coefficient and smaller average path length. Unlike the random rewiring rule in the WS model described by Watts and Strogatz, we use the "rich-gets-richer" rule that links a vertex that already has a large number of connections and has a higher probability. Simulation results also verify that the novel "rich-gets-richer" rule based small-world network is an improvement over the WS model.

1. Introduction

Networks have been widely used to model many real-life systems, such as biological systems, social interacting species, computer networks and so on. Vertices (nodes) of networks are the elements of the system and edges (links) represent the interactions between them [1]. In order to characterize the structural properties of the network, the connection topology is assumed to be either completely regular or completely random. However, the connection topology of some real biological and social networks lies somewhere between these two extremes.

To interpolate between regular and random networks, small-world networks were proposed by Watts and Strogatz [2] (WS model). In the WS model, edges are randomly rewired with a small uniform probability p based on a regular network. In order to prevent the formation of isolated clusters, which is possible in the WS model, an important variant of the WS model was proposed [4]. In the revised model, a small number of edges are directly added to set up connections between randomly chosen pairs of nodes without removal of edges from the regular networks. Other construction methods can also be found in [12,10]. It is interesting that small-world networks have been identified in many different fields, such as the World Wide Web, transportation systems, communication networks, the neuronal network, the electric power grid of southern California and so on [2,11,6–8].

The small-world network is characterized by two quantities: average path length L and clustering coefficient C. L is defined as being the average number of links in the shortest path between a pair of vertices in the network. C measures the probability that two nodes connected to a common vertex are also connected to each other [3]. Obviously, L measures the typical separation between two vertices in the network (a global property), whereas C measures the cliquishness of a typical neighbourhood (a local property). In the case of random networks, average path length L is short (i.e., $L \sim \log N$, where N is the number of nodes), but the network is poorly clustered (i.e., $C \ll 1$). On the contrary, regular networks are typically highly clustered, but L is comparable to the order of the network N. The small-world network interpolated between regular and random networks has the advantages of both random and regular networks. In other words, it has small L and large C.

The paper is organized as follows: In the next section, we introduce the basic model of small-world networks introduced by Watts and Strogatz [2] and our improved model of small-world networks. In Section 3, we carry out simulations to test the effect of our new rewiring rule. Finally, we present a summary.

2. "Rich-gets-richer" rule based small-world networks

It is well known that the models of small-world networks are constructed from a regular network by randomly rewiring each edge with a certain probability p. It is clear that the higher the probability, the more disordered the networks get (e.g., $p=1$, random networks). Small-world networks are obtained when

* Corresponding author.
E-mail address: hongweidai@hotmail.com (H. Dai).

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the rewiring probability is kept at a relatively small value (e.g., $p=0.005$).

Fig. 1 indicates the rewiring produced for interpolating between a regular ring lattice (Fig. 1(A)) and a random network (Fig. 1(C)). Fig. 1(B) shows the small-world network generated by the random rewiring rule. The small-world network generated by the "rich-gets-richer" rule is depicted in Fig. 1(B′).

Before introducing the new rewiring rule, random rewiring procedure in [2] will be introduced at first. Begin with a regular network consisting of N vertices arranged in a ring, and each connected to its k nearest neighbours. (In Fig. 1, $N=20$ and $k=4$.) Then a vertex is selected and also the edge that connects it to its nearest neighbour in a clockwise sense. With probability p, the selected edge would be rewired to a vertex chosen at random over the entire ring. It should be noticed that no vertex can be connected to itself and the different vertices have only one connection between them. This process will be repeated by moving clockwise around the ring considering each vertex in turn until one lap is completed. Then the edge that connects a vertex to its second nearest neighbour clockwise is selected and randomly reconnects the selected edge with probability p. This process will be continued, circulating around the ring and proceeding outward to more distance neighbours (Max$=k/2$) after each lap, until each edge in the original lattice has been considered once [2].

The main difference between our new model and the WS model lies in the selection of the vertex to be reconnected. As described in Fig. 1(B), the WS model rewires an edge to a vertex chosen randomly over the entire ring. For example, we choose vertex a and the edge that connects it to its nearest neighbour. If vertex a' is selected randomly to be connected to vertex a, then a long-range edge $E_{aa'}$ is generated. Similarly, a long-range edge $E_{bb'}$ can also be generated. In our new model, we use the "rich-gets-richer" rule that links a vertex that already has a large number of connections and has a higher probability. For example, in the first lap, vertex b and the edge that connects it to its nearest neighbour are selected. b' is the vertex chosen randomly to be connected to vertex b. According to the WS model, a long-range edge bb' will be added. Unlike the WS model, we define a vertex set V composed of b' and its $2r$ nearest neighbours on the original ring. That is $V=\{v_{b'-r},\ldots,v_{b'},\ldots,v_{b'+r}\}$. A selection pressure is added to vertex b' by comparing the degree D of b' with other vertices located in V. Here, degree is defined as the number of edges connected to a vertex. In Fig. 1, the vertex set radius r is set to $r=1$. Because $D_{a'}>D_{b'}$ (clearly, $D_{a'}=5$ and $D_{b'}=4$), instead of long-range edge bb', a new long-range edge ba' is added to the network.

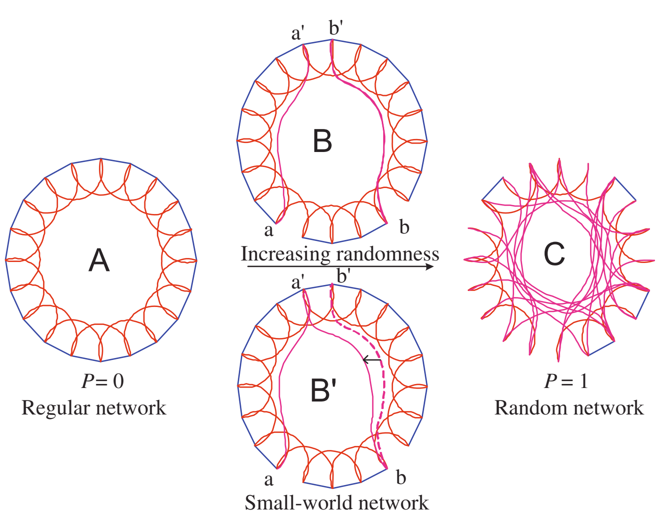

Fig. 1. Edge rewiring of the WS model and the "rich-gets-richer" rule based model.

It should be noticed that both the WS model and our new model add long-range edge without changing the number of vertices or edges of the network. Some variations of the WS model can be found in [4,5]. In these models, edges are added between randomly chosen pairs of nodes, but no edges are removed from regular lattice. Furthermore, growing small-world networks [9,10] have been introduced and studied recently.

3. Simulations

In this section, computational simulations are carried out to investigate the property of small-world network generated by the proposed novel rewiring rule.

As mentioned above, average path length L and clustering coefficient C are two main parameters to characterize the global and local properties. If d_{ij} indicate the shortest distance between vertex i and vertex j, the average path length L can be expressed as

$$L=\frac{\sum_{i=1}^{N}\left(\frac{\sum_{j=1,j\neq i}^{N} d_{ij}}{N-1}\right)}{N}=\frac{\sum_{i,j=1}^{N} d_{ij}}{N(N-1)} \tag{1}$$

Suppose that a vertex i has k_i neighbours, then the total possible number of edges $EP_i=C_{k_i}^2=k_i(k_i-1)/2$. If E_i denote the number of edges that actually exist, then clustering coefficient C_i of vertex i can be expressed as $C_i=E_i/EP_i$. Clustering coefficient C of the whole network can be calculated as the average of C_i over all vertices, namely,

$$C=\frac{\sum_{i=1}^{N} E_i/EP_i}{N}=\frac{\sum_{i=1}^{N} C_i}{N} \tag{2}$$

In the following, we show the average path length L and clustering coefficient C of small-world network generated by our novel rewiring rule, and also compare them with the WS model.

Fig. 2 shows the numerical computation of the average path length $L(p)$ and the clustering coefficient $C(p)$ of networks. In order to make a fair comparison between our improved model and the WS model, we carried out our simulation according to the conditions described in [2]. All the simulation results are averages over 20 random realizations of the rewiring process of the WS model and the new model, and have been normalized by the values $C(0)$ and $L(0)$ for a regular lattice. All the networks have

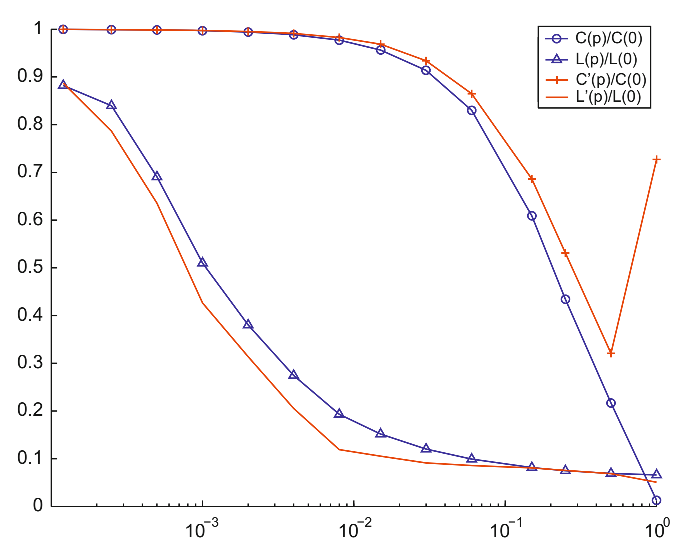

Fig. 2. Clustering coefficient and average path length for the WS model ($C(p)$, $L(p)$) and the new model ($C'(p)$, $L'(p)$).