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SCIENCE DIRECT
Robotics and Autonomous Systems 54 (2006) 559–566
Robotics and Autonomous Systems
www.elsevier.com/locate/robot

Online adaptation of reference trajectories for the control of walking systems

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Received 19 January 2005; received in revised form 20 March 2006; accepted 7 April 2006

Abstract

A simple and widely used way to make a robotic system walk without falling is to make it track a reference trajectory in one way or another, but the stability obtained this way may be limited and even small perturbations may lead to a fall. We propose here a series of heuristics to improve the stability that can be obtained from such a tracking control law, through an online adaptation of the choice of the reference trajectory being tracked. Encouraging simulations are obtained in the end on a simple planar biped model.

Keywords: Online adaptation; Parametrized trajectories; Constrained dynamics; Walking systems; Viability

1. Introduction

When a mechanical system is walking, it has many possible contacts with the ground which are regularly broken and recovered in order to produce a displacement of the whole structure. This allows traveling across obstacles with great versatility, but at the cost of a strong instability stemming from the fact that the dynamics of walking systems depends strongly on the forces that can be obtained from these contacts.

Being concerned with the stability of walking systems, this publication begins therefore in Section 2 with a general model of their dynamics that builds on a *unilateral* model of the contacts with the ground and allows us to specify which movements a walking system can do and which movements it can't [13,14]. The possibility to avoid to fall can be modeled then as a viability and invariance property.

A simple and widely used way to obtain such invariance properties is to make a walking system track a reference trajectory in one way or another [2–4,6–15], but the invariance obtained this way may be limited and even small perturbations may lead to a fall. How to improve the capacity for a walking system to avoid to fall has therefore been the main goal of most of the research done in the field of walking systems, and especially in the field of biped walking systems, since this stability issue is particularly problematic for them.

Since the problem is the availability of contact forces, it has been proposed to deal more properly with them by lowering the needs of the trajectory tracking, tracking for example trajectories with only some parts of the system [10,12] or allowing some deviations from the reference trajectories when forced to do so [4,6,10]. It has been proposed also to adapt the reference trajectory being tracked to the availability of contact forces, but most of such propositions so far don't clearly define when and how such an adaptation should occur, relying on parameters that need to be set with no clear relation to the global stability of the system [8,15]. A more radical approach is even to completely generate online the reference trajectories [2], but in a way, all of these approaches blur the effects of tracking reference trajectories, leading to an uncertain result as to really improving the capacity to avoid to fall.

Building on the analysis of Section 2, we propose here, in Section 3, a series of heuristics for the online adaptation of the reference trajectory being tracked which builds on a well delimited set of reference trajectories and a strict tracking which continuously keeps an eye on the available forces [14]. We show then in Section 4 how to apply these heuristics to a simple planar biped model, leading to encouraging numerical experiments in Section 5.

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0921-8890/$ - see front matter
doi:10.1016/j.robot.2006.04.007

2. The dynamics and stability of walking systems

2.1. Structure of the dynamics

Whatever the walking system being considered, planar or three-dimensional, with any number of legs with or without feet, its dynamics can be classically written as a set of Euler–Lagrange equations:

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + G(q) = T(q)u + C(q)^T \lambda \tag{1}$$

where $T(q)u$ are actuation forces and $C(q)^T \lambda$ contact forces.

As for any mechanical system that is able to move around, its configuration vector q has to account for two different informations, the shape of the system on the one side, its position and orientation in space on the other [6,11,13]. The shape of the system can be described by its joint positions, a vector q_1, and its position and orientation in space can be described by the position and orientation of a frame attached to one of its parts, leading to a vector q_2 of dimension 3 for planar systems, 6 for three-dimensional systems.

If we consider then the structure of the vector q:

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

we can split the dynamics (1) to exhibit the same structure:

$$\begin{bmatrix} M_1(q) \\ M_2(q) \end{bmatrix} \ddot{q} + \begin{bmatrix} N_1(q,\dot{q}) \\ N_2(q,\dot{q}) \end{bmatrix} \dot{q} + \begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix} = \begin{bmatrix} T_1(q) \\ 0 \end{bmatrix} u + \begin{bmatrix} C_1(q)^T \\ C_2(q)^T \end{bmatrix} \lambda \tag{2}$$

where the actuation forces don't appear in the lower part [6,11, 13]:

$$M_2(q)\ddot{q} + N_2(q, \dot{q})\dot{q} + G_2(q) = C_2(q)^T \lambda. \tag{3}$$

2.2. Contact forces

It appears then that for a walking system to realize a movement $q(t)$, Eq. (3) must be satisfied with appropriate contact forces. But the physics of contact is such that these forces have limitations: in the general case (no gluing, especially), contacting solids can push one another but they can't pull one another (which is referred to as the *unilaterality* of contacts), and friction between them is limited [6,7,9,13]. This can be expressed as a vector inequality on the amplitudes λ of the contact forces:

$$\mathcal{A}(\lambda) \le 0. \tag{4}$$

Considering this restriction of contact forces together with the lower part of the dynamics (3), a necessary condition for a walking system to realize a movement $q(t)$ is that there exist contact forces $\lambda(t)$ such that:

$$\begin{cases} M_2(q)\ddot{q} + N_2(q, \dot{q})\dot{q} + G_2(q) = C_2(q)^T \lambda \\ \mathcal{A}(\lambda) \le 0. \end{cases} \tag{5}$$

Note that this condition can be shown to be a complete generalization of more usual criteria such as the Center of Pressure or the Zero Moment Point criteria [14].

2.3. Impacts

Note also that when a part of a walking system lands on the ground, a sharp change of velocity may happen, an *impact* which can be modeled as an instantaneous event, especially in the case of purely rigid bodies [9,13]. This way, an instantaneous version of the Euler–Lagrange equations:

$$M(q)\left[\dot{q}_+ - \dot{q}_-\right] = C(q)^T \Lambda \tag{6}$$

relates the velocity of the system before and after the impact, $\dot{q}_-$ and $\dot{q}_+$, to impulsive contact forces $C(q)^T \Lambda$.

2.4. Avoiding to fall, a viability condition

Condition (5) shows that a walking system's ability to control its movements, and especially to keep its balance, is bound to the availability of appropriate contact forces: falling is a permanent threat then, and a threat for the integrity of both the walking system and its environment. Avoiding to fall should therefore be considered as an essential condition for walking systems, to be taken care of before any other goal.

Now, if we consider the set $\mathcal{F}$ of positions where the system is considered as having fallen (where a part of the system other than the feet is in contact with the ground, for example), avoiding to fall means avoiding to be in a position $q \in \mathcal{F}$. A *viability* condition [1,13] naturally comes out then:

Definition. A state $(q, \dot{q})$ is considered as *viable* if and only if the system is able to realize a movement $q(t)$ starting from this state that never gets inside the set $\mathcal{F}$.

A state $(q, \dot{q})$ is therefore either viable and the system is able to avoid to fall from it, or non-viable and the system cannot avoid to fall from it. This way, if we consider the *viability kernel*, the union of all viable states (Fig. 1), avoiding to fall means always staying inside this kernel, which should then be considered as the primary goal for every control law of walking systems.

This concept of viability appears to be very general: it applies to any type of locomotion, any structure of robot as long as falling is considered as an event that should be avoided. It is unfortunately of poor practical use since the complexity of the dynamics of walking systems is such that it is generally computationally impossible to verify whether a state is viable or not.

Numerous viable states can be sorted out though – equilibrium points, cyclic movements or trajectories leading to one of these – so that viability still seems to be an interesting concept to refer to in the analysis and design of control laws, as we will see in the next section.

3. Tracking reference trajectories

3.1. Adapting the choice of the reference trajectories

A simple and widely used way to make a walking system avoid to fall is to make it track a reference trajectory, in one way or another [2–4,6–15]. Indeed, if a trajectory and a