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# Control and simulation of a tensegrity-based mobile robot

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ABSTRACT

Tensegrity structures can provide a new approach to the construction of mobile robots with different shapes and properties that usual robots, wheeled or legged, do not have. Tensegrity are light, deformable structures that may be able to adapt their form to unconstrained environments. The main issue of this paper is twofold, first, to derive appropriate and general dynamic equations of motion to study the movement of such structures in the space; second to demonstrate, by means of simulation, that a tensegrity structure can execute any desired trajectory path by actuating some or all of its elements.



## 1. Introduction

Tensegrity is an abbreviation for *tensile integrity* which was coined by Fuller [5] in the early 60's. Tensegrity were created by people coming from the art community, Snelson [20], being rapidly applied to other disciplines such as in the architectural context, for structures such as geodesic domes, [4], or later in space engineering to develop deployable antennas, [23]. A general definition for a tensegrity was given by Pugh [18]:

> *A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space.*

Here, the compressive elements, struts, can not decrease their length while the tensile elements, cables, can not increase it. In fact, there may exist a third kind of element, namely a bar, which can not vary its length. An example of a tensegrity structure is given in Fig. 1.

**Fig. 1.** Example of tensegrity structure.

From an engineering point of view, tensegrity are a special class of structures whose elements may simultaneously perform the purposes of structural force, actuation, sense and feedback control. They have a very high resistance/weight coefficient and are easily deformable. In such kind of structures, theoretically, pulleys or other kind of actuators may stretch/shorten some of the constituting elements in order to substantially change their form with a little variation of the structure's energy. It has been demonstrated that tensegrity structures are very similar to cytoskeleton structures of unicellular organisms [8,9], some of which are known to move. They are also very similar to muscle-skeleton structures of high efficiency land animals that can reach speeds up to 60 mph. As reported by Timoshenko and Young [24], these beings incorporate tensional elements in their muscle-skeleton system such that they maintain the structure integrity while acting it, storing and distributing energy, [13].

Due to these similarities with such organisms, we think that tensegrity structures may be a good candidate to construct mobile robots with arbitrary forms and capable of self-deformation in order to adapt efficiently to the environment where they work. Up to now, tensegrity have been mainly used for static applications where the length of all members is kept constant and actuation is performed only to compensate for external perturbations. In the last decades the tensegrity framework has been also used to build deployable structures, although the tensegrity paradigm has not been fully exploited either. It is not since very recent years that we find some relevant works towards this goal: for instance, Aldrich [1] put together several simple tensegrity structures to build a redundant manipulator robot. Paul et al. [17] and Masic

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and Skelton [14] proposed different self-propelled tensegrity architectures to build mobile robots.

The purpose of this paper is twofold: first, to study the dynamic equations of motion for generic tensegrity that will allow us to control them and hence obtaining the desired movement; second, to demonstrate, by means of simulation, that we can theoretically perform any trajectory in the space with a tensegrity-based mobile robot. So we present a detailed study of the dynamic equations of motion for tensegrity thinking in using them to construct mobile robots.

The study of the static and dynamic characteristics of such structures has previously received some attention by the scientific community in other areas. Some analytic solutions to the static problem were given by Murakami and Nishimura [16] and Kebiche et al. [12], or more recently, a quite complete static analysis review was given by Hernàndez and Mirats-Tur [7]. The dynamics of tensegrity were first studied by Motro et al. [15]. Kanchanasaratool and Williamson [10] studied dynamic particle models while considering the bars to be massless; other studies, Skelton et al. [19] or Sultan et al. [21] consider mass on bars. Also non-linear models and their linearization have been considered by Murakami and Nishimura [16] or Sultan et al. [22]. All those studies consider statics and dynamics from a structural point of view, for example the behavior of a tensegrity dome under heavy winds, but have not considered the possibility of a tensegrity with self-motion capability. None of the cited studies has considered, as presented here, the six degrees of freedom in the space.

This paper is organized as follows. We first obtain, in Section 2, the general equations of motion for any tensegrity structure by using Euler–Lagrange formulation. This is done for a generic coordinates set in order to make evident the problem present when inverting the inertia matrix. We then particularize for a given set of coordinates in Section 3. Concretely, we analyze the classical Euler-angles representation and, later, we present the tensegrity motion equations using quaternions so as to avoid the inertia matrix inversion problem. Next, as we want to simulate the movement of a tensegrity-based robot, some analysis about ground and friction is performed in Section 4. This will be later used in the developed simulator, presented in Section 6. Section 5 hands in a particularization of the general equations of motion for the case of a 3-bar tensegrity-based prism. This will be the considered structure in Section 7 in order to study its control. Results comprising different trajectories followed by this simple tensegrity-based robot are reported in Section 8. Finally, the main conclusions of this work are outlined in Section 9.

## 2. General equations of motion for a tensegrity structure

Consider a generic tensegrity structure $\mathcal{T}$ with $b$ bars, and hence, $2b$ nodes, connected by a set of $c$ cables. Without loss of generality, cylindrical tubular bars are considered with internal end external radius $r_{1i}$ and $r_{2i}$, mass $m_i$, and length $l_i$. The purpose of this section is to obtain the general *Euler–Lagrange* equations of motion for such structures.

A general coordinates vector $\boldsymbol{q}_i^{\mathrm{T}} = (\boldsymbol{p}_i^{\mathrm{T}}, \boldsymbol{s}_i^{\mathrm{T}})$, is used to define the pose of the $i$th bar, where $\boldsymbol{p}_i$ contains the position of the bar's center of mass (3 orthogonal components) and $\boldsymbol{s}_i$ its orientation ($n$ components). For each bar, $\boldsymbol{q}_i^{\mathrm{T}}$ is independent and must have six degrees of freedom, although it is important to note that $\boldsymbol{q}_i^{\mathrm{T}}$ does not necessarily has six components. In general, $\boldsymbol{q}_i$ will have $m = 3 + n$ components and an associated vector of $n - 3$ constraints, $\boldsymbol{\Phi}(\boldsymbol{q}_i) = 0$.

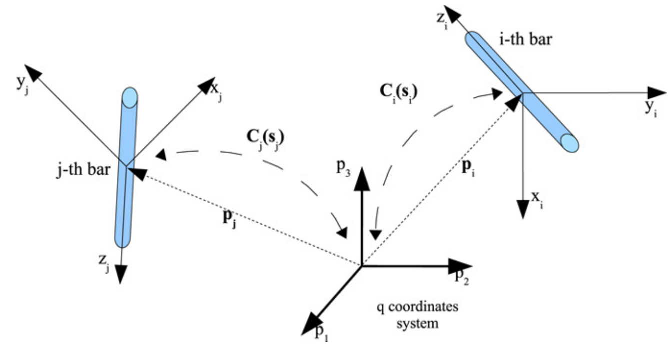


**Fig. 2.** Common and bar attached coordinate systems.

### 2.1. Kinetic energy

Let $T_i$ be the kinetic energy for the $i$th bar and $T$ the total kinetic energy for the tensegrity. Let $\boldsymbol{I}_i(\boldsymbol{q}_i)$ be the inertia matrix referenced to the $\boldsymbol{q}_i$ coordinates, $\boldsymbol{I}_i(\boldsymbol{q}_i) = \boldsymbol{C}_i(\boldsymbol{s}_i)^{\mathrm{T}}\boldsymbol{I}_i^{xyz}\boldsymbol{C}_i(\boldsymbol{s}_i)$, then $T_i$ is obtained as:

$$T_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) = \frac{1}{2}m_i\dot{\boldsymbol{p}}_i^{\mathrm{T}}\dot{\boldsymbol{p}}_i + \frac{1}{2}\dot{\boldsymbol{s}}_i^{\mathrm{T}}\boldsymbol{I}_i(\boldsymbol{s}_i)\dot{\boldsymbol{s}}_i \tag{1}$$

where $\boldsymbol{C}_i(\boldsymbol{s}_i)$ is the basis change matrix from the rotations associated to $\boldsymbol{q}_i$, formed by $\dot{\boldsymbol{s}}_i$, to a 3-D rotation basis adapted to the $i$th bar in which the third component is the longitudinal axis of the bar and the first two are orthogonal to it and between them (see Fig. 2 for clarity). That is, $\boldsymbol{C}_i$ is the matrix relating $\dot{\boldsymbol{s}}_i$ to the angular velocity of the bar, $\boldsymbol{\omega}_i = \boldsymbol{C}_i(\boldsymbol{s}_i)\dot{\boldsymbol{s}}_i$. On the other hand, $\boldsymbol{I}_i^{xyz}$ only depends on the physical parameters of the bar, where $r_i = r_{1i}^2 + r_{2i}^2$:

$$\boldsymbol{I}_i^{xyz} = \begin{pmatrix} \frac{1}{12}m_i(3r_i + l_i^2) & 0 & 0 \\ 0 & \frac{1}{12}m_i(3r_i + l_i^2) & 0 \\ 0 & 0 & \frac{1}{2}m_i r_i \end{pmatrix}. \tag{2}$$

In order to obtain $\boldsymbol{C}_i(\boldsymbol{s}_i)$, consider $\boldsymbol{S}_i$ the matrix of basis change from the bar-adapted basis to $\boldsymbol{p}_i$. As $\boldsymbol{S}_i$ is a rotation matrix and hence orthogonal,

$$\boldsymbol{S}_i^{\mathrm{T}}\dot{\boldsymbol{S}}_i = \begin{pmatrix} 0 & -\Omega_{i3} & \Omega_{i2} \\ \Omega_{i3} & 0 & -\Omega_{i1} \\ -\Omega_{i2} & \Omega_{i1} & 0 \end{pmatrix}. \tag{3}$$

Each of the $\Omega_{ij}$ can be expressed as, $\Omega_{ij}(\boldsymbol{s}_i) = \sum_{k=1}^{n} a_{jk}(\boldsymbol{s}_i)\dot{s}_{ik}$, so,

$$\boldsymbol{C}_i(\boldsymbol{s}_i) = \begin{pmatrix} a_{11}(\boldsymbol{s}_i) & a_{12}(\boldsymbol{s}_i) & \cdots & a_{1n}(\boldsymbol{s}_i) \\ a_{21}(\boldsymbol{s}_i) & a_{22}(\boldsymbol{s}_i) & \cdots & a_{2n}(\boldsymbol{s}_i) \\ a_{31}(\boldsymbol{s}_i) & a_{32}(\boldsymbol{s}_i) & \cdots & a_{3n}(\boldsymbol{s}_i) \end{pmatrix}. \tag{4}$$

Notice that $\boldsymbol{I}_i(\boldsymbol{q}_i)$ is symmetrical. Now, from (1) a new $m \times m$ matrix containing information about displacement and rotational energies can be defined,

$$\boldsymbol{M}_i(\boldsymbol{q}_i) = \begin{pmatrix} m_i\boldsymbol{I}^{3\times 3} & \boldsymbol{0}^{3\times n} \\ \boldsymbol{0}^{n\times 3} & \boldsymbol{I}_i(\boldsymbol{q}_i) \end{pmatrix} \tag{5}$$

and the kinetic energy for the $i$th bar is

$$T_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) = \frac{1}{2}\dot{\boldsymbol{q}}_i^{\mathrm{T}}\boldsymbol{M}_i(\boldsymbol{q}_i)\dot{\boldsymbol{q}}_i. \tag{6}$$