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Letters

Less conservative delay-dependent stability criteria for neural networks with time-varying delays ☆

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ABSTRACT

This paper is concerned with the stability for static neural networks with time-varying delays. With an appropriate Lyapunov functional formulated, a new technique is proposed to up bound the derivative of the Lyapunov functional. A delay-dependent stability criterion is obtained by proving the bound negative definite with convex combination methods. The delay-dependent stability criterion is simpler and less conservative than some existing ones. Both delay-independent and delay-dependent criteria are obtained, which can be checked easily using the recently developed algorithms. Examples are provided to illustrate the effectiveness and the reduced conservatism of the proposed results.

1. Introduction

During the last decade, recurrent neural networks (RNNs) have been extensively studied for their successful applications in signal processing, pattern recognition, associative memory, optimization problem and other engineering or scientific areas [1–3]. Since delays frequently occur in RNNs, and they are often a source of instability and oscillations, considerable attention has been devoted to stability for RNNs with delays, and some results on this topic have been reported in the literature. The stability results can be classified into two types, the delay-independent type (e.g. [3–7] and reference therein) and the delay-dependent type [27,28]. Delay-dependent stability results are generally less conservative than delay-independent ones, especially when the size of the delay is small.

Based on the difference in basic variables (local field states or neuron states), RNNs can be divided into static neural networks and local field networks [18]. For local neural networks, delay-dependent stability has been developed with considerable interests. When the neural network involves a constant delay, delay-dependent results can be found in [10,12–14]. For the case of time-varying delays delay-dependent criteria were reported in [8,9,15,16,29]. As far as distributed delays concerned, delay-dependent stability was addressed in [11,17].

Though the local neural network has been studied thoroughly, the static neural network has received little attention, with only a few stability results available. For the static neural network without delays, stability conditions were proposed in [19], when the connection weighting matrix is symmetric, while robust stability analysis was conducted in [20], where an LMI approach was employed. As for the neural networks with a constant delay, a Lyapunov functional method was developed to derive a delay-independent stability criterion [21]. Recently this method was extended to delay-dependent stability for static neural networks with time-varying delays. By constructing a Lyapunov functional and introducing relaxation matrices to estimate the derivative of the Lyapunov functional, a delay-dependent stability result was obtained in [30].

In this paper, attention is paid to the stability analysis for static neural networks with time-varying delays. Based on a new Lyapunov functional, we propose a novel technique to estimate the derivative of the Lyapunov functional without introducing a relaxation matrix. The obtained delay-dependent stability criterion involves few matrices but has less conservatism compared with some existing ones. It can be applied to any delay whose derivative is small or large or even unknown. A delay-independent criterion is also presented. Both delay-independent and delay-dependent criteria are expressed in LMIs; therefore they can be verified with the help of Matlab LMI toolbox. Examples are given to show the effectiveness and the reduced conservatism of the derived results.

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Notation: The notations in this paper are quite standard. For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension, and the superscript "T" represents the transpose, and the asterisk $*$ is used to denote a matrix that can be inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Main results

Consider the following recurrent network with time-varying delays:

$$\dot{u}(t) = -Au(t) + g(Wu(t-\tau(t)) + J)$$

$$u(t) = \psi(t), \quad -\tau \leq t \leq 0 \tag{1}$$

where

$$u(t) = [u_1(t)\ u_2(t)\ \cdots\ u_n(t)]^T$$

is the state vector associated with the n neutrons;

$$g(u(t)) = [g_1(u_1(t))\ g_2(u_2(t))\ \cdots\ g_n(u_n(t))]^T$$

represents the neuron activation function with $g(0) = 0$; $A = diag(a_1, a_2, \ldots, a_n) > 0$; W is the delayed connection weigh matrix; $J = [j_1\ j_2\ \cdots\ j_n]^T$ is a constant input from outside the system; the time delay $\tau(t)$ is a time-varying differentiable function satisfying

$$0 \leq \tau(t) \leq \tau$$

and

$$\dot{\tau}(t) \leq \mu;$$

$\psi(t)$, $-\tau \leq t \leq 0$, is the initial condition of system (1).

The following assumption will be made throughout the paper:

Assumption 1. The bounded neuron activation function in (1) satisfies

$$0 \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq l_i \quad (s_1 \neq s_2), \ \ i = 1, 2, \ldots, n$$

in which l_i $(i = 1, 2, \ldots, n)$ are known real constants.

Remark 1. The neural network in the form of (1) is a so-called static neural network model [18]. Common examples are the recurrent back-propagation neural networks [22], the optimization type networks [23] and the brain-state-in-a-box type networks [24]. Under the condition that W is invertible and $WA = AW$, by means of $y(t) = Wu(t) + J$ the static neural network can be transformed into the other kind, namely the local neural network

$$\dot{y}(t) = -Ay(t) + Wg(y(t-\tau(t))) + AJ \tag{2}$$

which has been extensively studied in the literature. However, many static neural networks do not meet this condition, as pointed out in [21].

Under Assumption 1, there exists an equilibrium point of (1) [2]. Let u^* be an equilibrium point of network (1). For simplicity, we make the following transformation

$$x(t) = u(t) - u*$$

to network (1). Then it becomes

$$\dot{x}(t) = -Ax(t) + f(Wx(t-\tau(t))) \tag{3}$$

$$x(t) = \varphi(t), \quad -\tau \leq t \leq 0$$

where $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ is the state vector of the transformed system (3); $\varphi(t) = \psi(t) - u^*$ is the initial condition, and

$$f(x(t)) = [f_1(x_1(t))\ f_2(x_2(t))\ \cdots f_n(x_n(t))]^T$$

with $f(x(t)) = g(x(t) + u^* + J) - g(u^* + J)$.

It is noted that $f(0) = 0$ and

$$0 \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq l_i \quad (s_1 \neq s_2), \ \ i = 1, 2, \ldots, n. \tag{4}$$

Obviously neural network (3) admits an equilibrium point $x(t) \equiv 0$ corresponding to the initial condition $\varphi(t) \equiv 0$, $-\tau \leq t \leq 0$.

The stability analysis problem addressed in this paper is to establish some conditions, under which the origin of neural network (3) is globally asymptotically stable.

To solve this problem, we need the following lemma:

Lemma 1. Gu ([25]) *For any symmetric positive definite matrix* $M > 0$, *scalar* $\gamma > 0$ *and vector function* $\omega : [0, \gamma] \to R^n$ *such that the integrations concerned are well defined, the following inequality holds*:

$$\left(\int_0^\gamma \omega(s)\, ds \right)^T M \left(\int_0^\gamma \omega(s)\, ds \right) \leq \gamma \left(\int_0^\gamma \omega(s)^T M \omega(s)\, ds \right)$$

The following theorem provides a solvability condition for the stability analysis problem:

Theorem 1. *For given* μ *the origin of neural network* (3) *with* (4) *is globally asymptotically stable, if there exist matrices* $P > 0$, $Q \geq 0$, $Q_1 \geq 0$ *and non-negative diagonal matrices D, T and S such that the following LMI holds*:

$$\Phi = \begin{bmatrix} -PA - AP + Q_1 & \Phi_{12} & P & 0 \\ * & \Phi_{22} & SW & 0 \\ * & * & \Phi_{33} & T\Sigma W \\ * & * & * & -(1-\mu)Q_1 \end{bmatrix} < 0 \tag{5}$$

where

$$\Sigma = diag(l_1, l_2, \ldots, l_n)$$

$$\Phi_{12} = -AW^T S + W^T \Sigma D$$

$$\Phi_{22} = Q - 2D$$

$$\Phi_{33} = -(1-\mu)Q - 2T$$

Proof. Let $x_t(s) = x(t+s)$, $-\tau \leq s \leq 0$, and

$$S = diag(s_1, s_2, \ldots, s_n).$$

Introduce a Lyapunov functional candidate for neural network (3) as

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{6}$$

where

$$V_1(x_t) = x(t)^T P x(t) + \int_{t-\tau(t)}^t x(\alpha)^T Q_1 x(\alpha)\, d\alpha \tag{7}$$

$$V_2(x_t) = \int_{t-\tau(t)}^t f(Wx(s))^T Q f(Wx(s))\, ds \tag{8}$$

$$V_3(x_t) = 2 \sum_{i=1}^n s_i \int_0^{W_i x(t)} f_i(s)\, ds \tag{9}$$

with W_i denoting the ith row of matrix W.