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Letters

Exponential stability of hybrid stochastic neural networks with mixed time delays and nonlinearity ☆

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A B S T R A C T

This paper is concerned with the problem of robust exponential stability for a class of hybrid stochastic neural networks with mixed time-delays and Markovian jumping parameters. In this paper, free-weighting matrices are employed to express the relationship between the terms in the Leibniz–Newton formula. Based on the relationship, a linear matrix inequality (LMI) approach is developed to establish the desired sufficient conditions for the mixed time-delays neural networks with Markovian jumping parameters. Finally, two simulation examples are provided to demonstrate the effectiveness of the results developed.

1. Introduction

Neural networks (cellular neural networks, Hopfield neural networks and bi-directional associative memory networks) have been intensively studied over the past few decades and have found application in a variety of areas, such as image processing, pattern recognition, associative memory and optimization problems [1–3]. In reality, time-delay systems are frequently encountered in various areas, e.g. in neural networks, where a time delay is often a source of instability and oscillations. Recently, both delay-independent and delay-dependent sufficient conditions have been proposed to verify the asymptotical or exponential stability of delay neural networks, see e.g. [4–10].

On the other hand, stochastic modeling has come to play an important role in many real systems [11,12], as well as in neural networks. Neural networks have finite modes, which may jump from one to another at different times. Recently, it has been shown in [13–14] that, the jumping between different neural network modes can be governed by a Markovian chain. Furthermore, in real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain stochastic inputs [15]. Hence, the stability analysis problem for stochastic neural networks becomes increasingly significant, and some results related to this problem have recently been published, see e.g. [15–17]. To the best of the authors' knowledge, the robust exponential stability analysis problem for uncertain stochastic neural networks with mixed time-delays and Markovian jumping parameters, which is still an open problem, has not yet been fully investigated.

In this paper, we study the global exponential stability problem for a class of hybrid stochastic neural networks with mixed time-delays and Markovian jumping parameters, where the mixed delays comprise discrete and distributed time-delays, the parameter uncertainties are norm-bounded, and the neural networks are subjected to stochastic disturbances described in terms of a Brownian motion. By utilizing a Lyapunov–Krasovskii functional candidate and using the well-known S-procedure, we convert the addressed stability analysis problem into a convex optimization problem. In this letter, the free-weighting-matrix approach is employed to derive a linear matrix inequality (LMI)-based delay-dependent exponential stability criterion for neural networks with mixed time-delays and Markovian jumping parameters. Note that LMIs can be easily solved by using the Matlab LMI toolbox, and no tuning of parameters is required.

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Numerical examples demonstrate the effectiveness of this method.

Notation: The notations in this paper are quite standard. R^n and $R^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the transpose and the notation $X\geq Y$ (respectively, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. Let $h>0$ and $C([-h,0];R^n)$ denote the family of continuous functions φ from $[-h,0]$ to R^n with the norm $|\varphi|=\sup_{-h\leq\theta\leq 0}|\varphi(\theta)|$, where $|.|$ is the Euclidean norm in R^n. A is a matrix, denoted by $|A|$ its operator norm, i.e., $|A|=\sup\{|Ax|: |x|=1\}=\sqrt{\lambda_{\max}(A^TA)}$, where $\lambda_{\max}(.)$ (respectively, $\lambda_{\min}(.)$) means the largest (respectively, smallest) eigenvalue of A. $l_2[0,\infty)$ is the space of square integrable vector. Moreover, let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_t\geq 0,P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_t\geq 0$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathscr{F}_0}([-h,0]);R^n)$ the family of all $\mathscr{F}_0$-measurable $C([-h,0];R^n)$-valued random variables $\xi=\{\xi(\theta):-h\leq\theta\leq 0\}$ such that $\sup_{-h\leq\theta\leq 0}E|\xi(\theta)|^p<\infty$, where $E\{.\}$ stands for the mathematical expectation operator with respect to the given probability measure P. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise. Finally, we use the symbol $maddt(X)$ to represent $X+X^T$.

2. Problem formulation

In this letter, the neural network with mixed time-delays is described as follows:

$$\dot{u}(t)=-Au(t)+W_0g_0(u(t))+W_1g_1(u(t-h))+W_2\int_{t-\tau}^{t}g_2(u(s))\,ds+V,\tag{1}$$

where $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]^T\in R^n$ is the state vector associated with n neurons and the diagonal matrix $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ has positive entries $a_k>0$. $W_0=(w^0_{ij})_{n\times n}$, $W_1=(w^1_{ij})_{n\times n}$ and $W_2=(w^2_{ij})_{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix, and the distributively delayed connection weight matrix. $g_k(u(t))=[g_{k1}(u_1),g_{k2}(u_2),\ldots,g_{kn}(u_n)]^T$ $(k=0,1,2)$ denotes the neuron activation function with $g_k(0)=0$, and $V=[V_1,V_2,\ldots,V_n]^T$ is a constant external input vector. The scalar $h>0$, which may be unknown, denotes the discrete time delay, where the scalar $\tau>0$ is the known distributed time-delay.

Assumption 1. The neuron activation functions $g_i(.)$ in (1), are bounded and satisfy the following Lipschitz condition

$$|g_k(x)-g_k(y)|\leq|G_k(x-y)|,\quad\forall x,y\in R\ (k=0,1,2),\tag{2}$$

where $G_k\in R^{n\times n}$ are known constant matrices.

Remark 1. In this letter, none of the activation functions are required to be continuous, differentiable and monotonically increasing. Note that the types of activation functions in (2) have been used in many papers, see [10,13,15–17].

Let u^* be the equilibrium point of (1). For the purpose of simplicity, we transform the intended equilibrium u^* to the origin by letting $x=u-u^*$, and then the system (1) can be transformed into:

$$\dot{x}(t)=-A(x)+W_0l_o(x(t))+W_1l_1(x(t-h))+W_2\int_{t-\tau}^{t}l_2(x(s))\,ds,\tag{3}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in R^n$ is the state vector of the transformed system. It follows from (2) that the transformed neuron activation functions $l_k(x)=g_k(x+u^*)-g_k(u^*)$ $(k=0,1,2)$ satisfy

$$|l_k(x)|\leq|G_k(x)|,\tag{4}$$

where $G_k\in R^{n\times n}$ $(k=0,1,2)$ are specified in (2).

By the model (3), we are in a position to introduce the hybrid stochastic neural networks with mixed time delays and nonlinearity as follows.

Let $\{r(t),t\geq 0\}$ be a right-continuous Markov process on the probability space, which takes values in the finite space $S=\{1,2,\ldots,N\}$ with generator $\Gamma=(\pi_{ij})(i,j\in S)$ given by

$$P\{r(t+\varDelta)=j|r(t)=i\}=\begin{cases}\pi_{ij}\varDelta+o(\varDelta) & \text{if } i\neq j,\\ 1+\pi_{ii}\varDelta+o(\varDelta) & \text{if } i=j,\end{cases}$$

where $\varDelta>0$ and $\lim_{\varDelta\to\infty}o(\varDelta)/\varDelta=0$. $\pi_{ij}\geq 0$ is the transition rate from i to j if $i\neq j$ and $\pi_{ii}=-\sum_{j\neq i}\pi_{ij}$.

We consider the following hybrid stochastic neural networks with mixed time delays and nonlinearity, which is actually a modification of (3).

$$\begin{aligned}dx(t)=\Big[&-(A(r(t))+\Delta A(r(t)))x(t)+(W_0(r(t))\\&+\Delta W_0(r(t)))l_0(x(t))+(W_1(r(t))+\Delta W_1(r(t)))l_1(x(t-h))\\&+(W_2(r(t))+\Delta W_2(r(t)))\int_{t-\tau}^{t}l_2(x(s))\,ds\Big]dt\\&+\sigma(t,x(t),x(t-h),r(t))\,d\omega(t).\end{aligned}\tag{5}$$

For notational convenience, we give the following definitions:

$$dx(t)=y(t,i)\,dt+\sigma(t,x(t),x(t-h),r(t))\,d\omega(t)\tag{6}$$

where

$$\begin{aligned}y(t,i)=\Big[&-(A(r(t))+\Delta A(r(t)))x(t)+(W_0(r(t))+\Delta W_0(r(t)))l_0(x(t))\\&+(W_1(r(t))+\Delta W_1(r(t)))l_1(x(t-h))+(W_2(r(t))\\&+\Delta W_2(r(t)))\int_{t-\tau}^{t}l_2(x(s))\,ds\Big]dt,\end{aligned}$$

$\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_m(t)]^T\in R^m$ is a Brownian motion defined on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$. Here,

$$\begin{aligned}&\Delta A(r(t))=M_A(r(t))F(t,r(t))N_A(r(t)),\\&\Delta W_k(r(t))=M_k(r(t))F(t,r(t))N_k(r(t)),\quad k=0,1,2,\end{aligned}\tag{7}$$

where $\Delta A(r(t))$ is a diagonal matrix, and $M_A(r(t))$,$N_A(r(t))$, $M_k(r(t))$, $N_k(r(t))$ $(k=0,1,2)$, are known real constant matrices with appropriate dimensions at mode $r(t)$. The matrix $F(t,r(t))$, which may be time-varying, is unknown and satisfies

$$F^T(t,r(t))F(t,r(t))\leq I,\quad\forall t\geq 0;\ \ r(t)=i\in S.\tag{8}$$

Assume that $\sigma:R^+\times R^n\times R^n\times S$ is local Lipschitz continuous and satisfies the linear growth condition [16]. Moreover, σ satisfies

$$\begin{aligned}&trace[\sigma^T(t,x(t),x(t-h),r(t))\sigma(t,x(t),x(t-h),r(t))]\\&\quad\leq|\Sigma_{1,r(t)}x(t)|^2+|\Sigma_{2,r(t)}x(t-h)|^2\end{aligned}\tag{9}$$

where Σ_{1i} and Σ_{2i} are known constant matrices with appropriate dimensions. Observe the system (5) and let $x(t;\xi)$ denote the state trajectory from the initial data $x(\theta)=\xi(\theta)$ on $-h\leq\theta\leq 0$ in $L^2_{\mathscr{F}_0}([-h,0];R^n)$. Clearly, the system (5) admits an equilibrium point (trivial solution) $x(t;0)\equiv 0$ corresponding to the initial data $\xi=0$. For all $\delta\in[-d,0]$, suppose that $\exists\varpi>0$, such that

$$|x(t+\delta)|\leq\varpi|x(t)|,\quad d=\max\{\tau,h\}.\tag{10}$$

Recall that the Markovian process $\{r(t),t\geq 0\}$ takes values in the finite set $S=\{1,2,\ldots,N\}$. For the sake of simplicity,