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ScienceDirect
Neurocomputing 71 (2008) 1857–1865
NEUROCOMPUTING
www.elsevier.com/locate/neucom

An approach for directly extracting features from matrix data and its application in face recognition

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Available online 29 February 2008

Abstract

By formulating two-dimensional principle component analysis (2DPCA) as a mathematical form different from the conventional 2DPCA, we present theoretical basis of 2DPCA and show the theoretical similarities and differences between 2DPCA and PCA. We also show that 2DPCA owns its decorrelation property and the feature vectors extracted from matrices are uncorrelated. We use the proposed mathematical form to show that 2DPCA is the best approach for directly extract features from matrices. We also present in detail 2DPCA Schemes 1 and 2, two schemes to implement the proposed mathematical form. The two schemes transform original images into different spaces, respectively, 2DPCA Scheme 1 enhances the transverse characters of images, whereas the second scheme enhances vertical characters of images. We propose a feature fusion approach for achieving better recognition results by combining the features generated from the two schemes of 2DPCA. The proposed fusion approach is tested on face recognition tasks and is found to be more accurate than both 2DPCA Scheme 1 and 2DPCA Scheme 2.

Keywords: Principal component analysis; Feature extraction; Feature fusion; Face recognition

1. Introduction

Principal component analysis (PCA) [1–10] is a widely applied dimension reduction and feature extraction technique. It has been used in handprint recognition [7], the recognition of man-made objects [6], industrial robotics [18], and image-based recognition systems [4,13]. PCA is generally implemented on image data as follows: first an image matrix is converted into a vector by concatenating its columns or rows. Then eigenvectors of the covariance matrix or correlation matrix of these vectors are used as transforming axes to obtain their principal components. PCA has been shown to be effective [4,5,9,10,13,14,18,19,21–24,28] but it does suffer from two particular problems. First, if the number of training samples is small and the data are high-dimensional, it is difficult to accurately estimate the covariance (or correlation) matrix. Second, because the one-dimensional vector space derived from images is usually of very large dimensionality, implementation of PCA is usually very time consuming [24,25].

Responding to these drawbacks, two-dimensional PCA (2DPCA), a novel transform technique derived from the PCA technique, directly extracts features from image matrices [25,26]. Note that 2DPCA as the generation matrix takes the covariance matrix (or correlation matrix) of the image matrix rather than the corresponding one-dimensional vector. 2DPCA calculates directly the projection of a matrix onto the transforming axis. 2DPCA is much more efficient than PCA [25], requiring less memory and having a lower computational cost and has obtained promising experimental results in the areas of feature extraction and dimension reduction. A further difference between traditional PCA and 2DPCA is that in PCA every feature that is extracted is a scalar whereas in 2DPCA every extracted feature is a vector, hereafter called a feature vector. However, it is not known whether the approach is theoretically well-founded.

In this paper we will analyze the theoretical basis of 2DPCA and propose a new 2DPCA-based feature fusion

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doi:10.1016/j.neucom.2007.09.021

approach that combines the feature extraction results of two 2DPCA implementation schemes. We will further show that under the proposed mathematical form of 2DPCA, certain fine theoretical properties hold, for example decorrelation.

Some previous literatures also provide valuable investigation of 2DPCA. For example, Wang et al. [15–17] demonstrated that 2DPCA was equivalent to a special case of block-based PCA [3]. Xu et al. [20] constructed two transformation matrices based on the 2DPCA technique, and performed two transforms to obtain features of a matrix. Motivated by the successes of the two-dimensional linear discriminant analysis, Tao et al. [11,12] developed a general tensor discriminant analysis. Ye et al. proposed generalized principal component analysis (GPCA) in [27]. GPCA also works directly with images in their native state, as two-dimensional matrices, by projecting the images to a vector space that is the tensor product of two lower-dimensional vector spaces.

The differences between our 2DPCA schemes and the previous approaches are as follows. Liwei Wang's approach [15,17] seems to be computationally equivalent to our 2DPCA scheme 1. However, he also did not analyze the theoretical basis of 2DPCA whereas we analyze this and indicate the decorrelation property of 2DPCA. Different from Xu's approach [20] of consecutively performing two transforms to obtain features of a matrix, this paper focuses on fusing two classes of image features obtained using two different implementation schemes of 2PCA to improve face recognition performance. That is, the transform matrices generated from 2DPCA Scheme 1 and 2DPCA Scheme 2 were first used to transform image matrices into two classes of features. Then the two classes of features were fused for face recognition by using a matching score fusion approach. Although from the point of view of methodology, GPCA [27] and 2DPCA belong to the same class of technique, that is, they can both extract features directly from a two-dimensional matrix, no closed form solution exists for GPCA and it cannot be proved theoretically that the two 2DPCA schemes presented in this paper are special cases of GPCA.

The rest of the paper is organized as follows. Section 2 formally presents 2DPCA and its theoretical basis and introduces two 2DPCA implementation schemes. Section 3 presents the characteristics of the reconstruction images, respectively, associated with the two implementation schemes. Section 4 proposes the 2DPCA-based feature fusion approach. Section 5 offers a brief Conclusion.

2. Theoretical basis and implementation schemes of 2DPCA

Suppose there are M images and $A_1, A_2, \ldots, A_M$ are, respectively, the matrices corresponding to these images. The conventional 2DPCA [25] as the generation matrix takes the following covariance matrix:

$$G_t = \frac{1}{M}\sum_{i=1}^{M}((A_i - \bar{A})^{\mathrm{T}}(A_i - \bar{A})),$$

where $\bar{A}$ is the mean of all the image matrices. Eigenvalues and eigenvectors of G_t should be first determined, and then k eigenvectors associated with the k largest eigenvalues are selected as transforming axes. The conventional 2DPCA projects an image matrix onto these transforming axes, respectively, and regards the resultant k projections (k vectors) as features of the image [25]. One of the advantages of 2DPCA is that it is much more efficient than PCA [25]. In addition, 2DPCA has obtained promising experimental results in the areas of feature extraction and dimension reduction. However, it is not known whether the approach is theoretically well-founded.

2.1. 2DPCA Scheme 1

In this paper, 2DPCA Scheme 1 is referred to as the 2DPCA technique based on the generation matrix $\Sigma_1 = E(A^{\mathrm{T}}A)$, where A stands for a two-dimensional matrix. Note that the generation matrix of 2DPCA Scheme 1 is formally different from that of the conventional 2DPCA, thus we say that 2DPCA Scheme 1 formulates the 2DPCA technique as a new mathematical form. This subsection will address this issue of whether the 2DPCA technique is theoretically well-founded. Indeed, our analysis will demonstrate that 2DPCA Scheme 1 is able to produce the minimal reconstruction error and uncorrelated feature vectors. Suppose that non-decreasing eigenvalues of Σ_1 are $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n$. 2DPCA Scheme 1 takes the r eigenvectors corresponding to the first r largest eigenvalues of Σ_1 as transforming axes to directly extract features from a matrix. Using 2DPCA Scheme 1, we can project a matrix onto a transforming axis to produce a feature vector (column-feature-vector). If 2DPCA Scheme 1 exploits multiple transforming axes for feature extraction, the feature extraction results will be multiple column-feature-vectors, which can form a new matrix. In this sense, we say that 2DPCA Scheme 1 transforms an original matrix into a new matrix with smaller dimension. We begin with the following theorem to analyze theoretical basis of 2DPCA Scheme 1.

Theorem 1. *Measured using mean squared error, 2DPCA is the best technique for directly transforming matrices into feature vectors as feature vectors obtained using the 2DPCA technique allow matrices to be reconstructed with the minimum mean-square reconstruction error.*

Proof. Suppose that image matrix A can be accurately expressed in terms of

$$A = \sum_{i=1}^{n} v_i u_i^{\mathrm{T}}, \quad 1 \leqslant i, \quad j \leqslant n, \tag{1}$$

where

$$u_i^{\mathrm{T}} u_j = \begin{cases} 1 & i = j, \\ 0 & i \neq j, \end{cases} \quad u_i\,(1 \leqslant i \leqslant n)$$