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Models of human movement: Trajectory planning and inverse kinematics studies

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ABSTRACT

The seemingly simple everyday actions of moving limb and body to accomplish a motor task or interact with the environment are incredibly complex. To reach for a target we first need to sense the target's position with respect to an external coordinate system; we then need to plan a limb trajectory which is executed by issuing an appropriate series of neural commands to the muscles. These, in turn, exert appropriate forces and torques on the joints leading to the desired movement of the arm. Here we review some of the earlier work as well as more recent studies on the control of human movement, focusing on behavioral and modeling studies dealing with task space and joint-space movement planning. At the task level, we describe studies investigating trajectory planning and inverse kinematics problems during point-to-point reaching movements as well as two-dimensional (2D) and three-dimensional (3D) drawing movements. We discuss models dealing with the two-thirds power law, particularly differential geometrical approaches dealing with the relation between path geometry and movement velocity. We also discuss optimization principles such as the minimum-jerk model and the isochrony principle for point-to-point and curved movements.

We next deal with joint-space movement planning and generation, discussing the inverse kinematics problem and common solutions to the problems of kinematic redundancy. We address the question of which reference frames are used by the nervous system and review studies examining the employment of kinematic constraints such as Donders' and Listing's laws. We also discuss optimization approaches based on Riemannian geometry.

One principle of motor coordination during human locomotion emerging from this body of work is the intersegmental law of coordination. However, the nature of the coordinate systems underlying motion planning remains of interest as they are related to the principles governing the control of human arm movements.

1. Introduction

Successful performance of a motor task, like reaching for a cup of coffee, requires a series of sensory information processing and motion planning operations. The nature of these processes is not well understood. For example, it is not entirely clear whether and how the nervous system generates motion plans and, if so, at what level these plans are constructed. This issue is still being strongly debated, but the motor system is generally considered to be hierarchically organized with movement generation represented at several levels—neural commands, muscle activations, joint motions, hand trajectories and task goals (Fig. 1). By contrast, in robotics, there is a consensus that the levels of movement planning and specification to be considered are the task (end-effector), configuration (joint) and actuator levels.

Regardless of whether movement plans are indeed represented in the nervous system, or at what level, it appears that in the forward direction a specific cortical and subcortical command pattern gives rise to specific muscle activation patterns, leading to movement and achievement of the task. The reverse direction, however, cannot be uniquely resolved. For each hand path, there are infinitely many possible velocity profiles and, in turn, many different joint rotations that achieve the same goal. Thus, specifying a pattern of behavior at any level completely specifies the patterns at the level below it (many-to-one) but the pattern will be consistent with many possible patterns and solutions at the levels above (one-to-many) [1]. Understanding how the central nervous system resolves such problems of redundancy and selection is one of the challenges addressed by both experimental and modeling studies of motor control and sensorimotor integration.

Here we review trajectory formation at the end-effector level, models dealing with motion planning at the joint level and the topic of intersegmental coordination during upper limb and locomotion tasks. We shall also discuss kinematic redundancy and timing. For models dealing with movement dynamics the reader is referred to other papers in the current issue.

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Fig. 1. An illustration of the motor hierarchy: The configuration of each level of the hierarchy uniquely determines the state of the level below (many-to-one). In the other direction, for each configuration in one space there are infinitely many configurations in the space above which satisfy it.

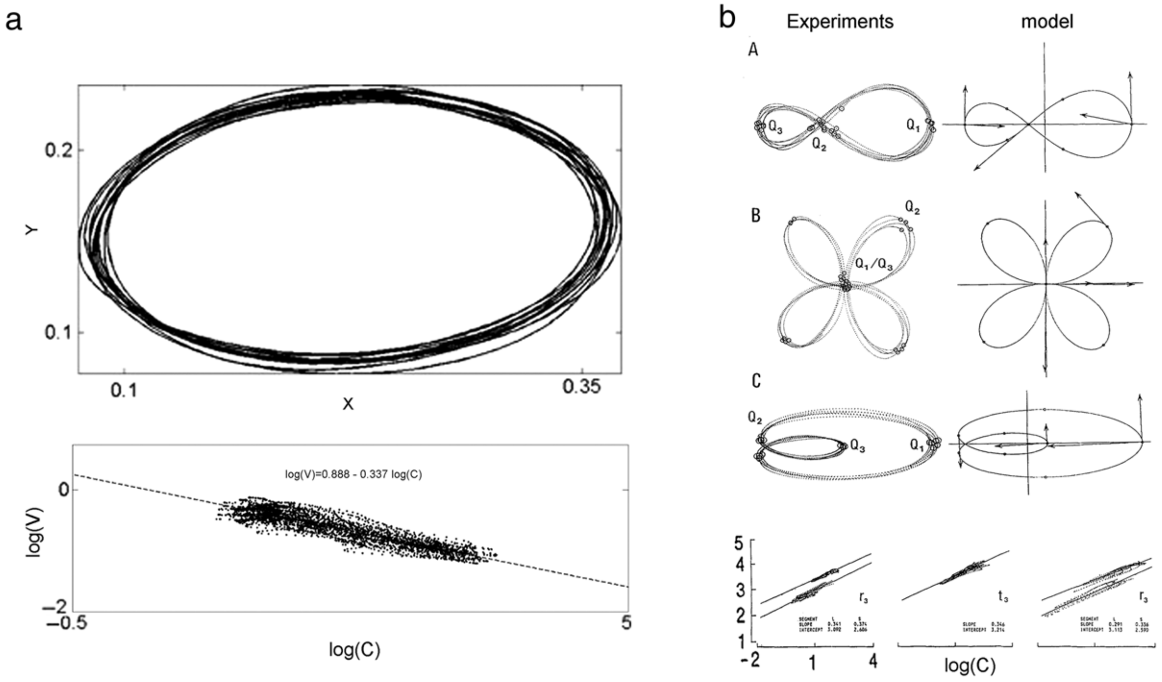

Fig. 2. Adapted from Viviani and Flash [19] to demonstrate the two-thirds power law. The left side illustrates the original observation of the two-thirds power law during drawing an ellipse (top) as demonstrated by the **log**(**v**) vs. **log**(**c**) plot (bottom), where **v** and **c** stand for velocity and curvature, respectively. The dashed line is the result of the linear regression between **log**(**v**) and **log**(**c**) and its slope is exactly minus one-third as predicted by the two-third power law. On the right, an extension of the two-thirds power law is demonstrated for more complex movements: figure-eight (A), limaçon (B) and cloverleaf (C). The experimental results are shown on the left and on the right the same shapes are presented as predicted by the minimum jerk-model, using boundary conditions that are depicted by the vectors. In the bottom of the figure **log**(**v**) vs. **log**(**c**) plots reveal an implied segmentation which is evident from the multiple straight lines with the same slope (minus one-third) but different velocity gain factors.
Source: Adapted from Viviani and Flash [19].

Influential theories on how the central nervous system (CNS) plans and generates movement were proposed, among others, by Lashley [2] and Bernstein [3]. However, only since the 80's has evidence accumulated that well defined formation principles operating in both space and time lead to the movements observed in a variety of motor tasks [4–8]. Many researchers have searched for the basic movements that humans can perform in both voluntary and constrained settings ([3], for a review see [9]), with subjects either instructed to carry out a well-defined task (e.g., 'trace an ellipse') or to carry out more natural behavioral tasks (e.g., 'slicing bread'). More recently, the idea that more complex movements are constructed from simpler building blocks, so-called motor or motion primitives, has emerged as the dominant approach [9,10]. Largely driven by the functional role of movements in numerous behavioral tasks, much effort has been devoted to decomposing these movements into their motor primitives, with success in both vertebrates and invertebrates [9]. Both animal and human motor studies have addressed the extraordinary abilities of nervous systems to generate complicated motion patterns by selecting among an enormous set of motor commands that are equivalent in terms of the motor task they subserve ("the redundancy problem" or "motor equivalence problem" [2,3]).

We first discuss several studies dealing with motor primitives and the empirically derived two-thirds power law [11] that links movement kinematics and geometry. Next, we present several theoretical studies focusing on geometrical aspects of motor control, highlighting the usefulness of geometrical invariance for resolving issues related to task space redundancy. Finally, an accumulating body of evidence has shown that motor regularities imposed by geometrical invariance are deeply related to optimization theory, the optimization principle being referred to here is the maximization of motion smoothness.

2. Kinematic features and models of hand trajectories

Many studies of human motor control have investigated the mechanisms underlying the control of the movement kinematic output, particularly the temporal and spatial characteristics of the end-effector (e.g., hand) trajectory. The discovery that curvature and speed co-vary in various motor tasks has been the focus of many detailed studies [11–17], eventually converging to the "two-thirds power law" [5,18–21]. This rule states that the angular velocity during movement is piecewise proportional to the path's curvature raised to the power of two-thirds. Due to its robust properties, this principle is now a well-recognized kinematic regularity in the movements of humans and other primates.

The coupling between curvature and speed in human movements, typically an inverse relation of speed to curvature, was established at the end of the 19th century [12]. Almost a hundred years later, this phenomenon was quantitatively formalized [5]. Using the tangential velocity v and path curvature C this principle can be expressed as:

$$|v(t)| = \alpha C^{-\frac{1}{3}}(t) \tag{1}$$

where α is piecewise constant and is called the velocity gain factor. Fig. 2(A) demonstrates this principle for the drawing of an ellipse