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Walking control for a planar biped robot using 0-flat normal form

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ABSTRACT

This paper deals with the use of 0-flat normal form to control a 7 d.o.f-biped robot to follow a specified trajectory. Sufficient geometrical conditions are given to transform the studied nonlinear systems into a 0-flat normal form and determine the flat outputs. On the other hand, a controller design strategy is proposed to control the walking robot. Simulations are carried out using Matlab. The results obtained are very convincing and show the usefulness of such a method in studying highly non-linear systems and designing control laws to drive them.

1. Introduction

Many biped robots have been developed in recent years and many biped robot control algorithms have been presented. A survey of the biped robot control literature enables us to classify several control design approaches into three classes.

The first class is based on intuitive control, it includes those approaches which use physical intuitive control strategies [1]. In the second class the control approach is based on intelligent learning control, it includes fuzzy logic controllers [2] and neural network controllers [3]. The third class is based on pre-computed reference trajectories, and an appropriate controller for geometric tracking of theses trajectories. Within the context of tracking many different control methods have been explored, including optimal control [4], computed torque control [5] and tracking control [6]. The problem of making a biped have a dynamically stable walk is interesting due to the complexity of the model:

- Bipeds are typically high degrees of freedom mechanical systems, that is many links and joints must be coordinated to achieve locomotion.
- Their hybrid nature resulting from the unavoidable impacts with the ground. These impacts produce discontinuities in the velocity vector.
- Their variable structure; indeed the state dimension varies from one walking phase to an other.

In fact, the problem can be considerably simplified if the system can be shown to have a differential flatness property. To the best knowledge of the authors, the few flatness-based control of biped robots concern simplified models (2 d.o.f) specific configurations only [7,8].

In this paper we aim at using the concept of differential flatness in order to develop a new methodology in tracking control for robotics systems. This concept was first addressed by Fliess and coworkers in [9]. It is a structural property of a class of nonlinear systems, for which, all states and inputs can be determined from flat outputs and their derivatives. Flatness has been studied by several mathematical frameworks such as differential algebra [9–11] and finite dimensional differential geometry [12–14]. Differential flatness has been successfully applied in planning and control to diverse engineering systems and differentially flat systems are presented with and without auxiliary constraints [9,15,16]. The book by Sira-Ramirez and Agrawal summarizes diverse engineering applications [17], see also [10] for a somewhat different approach. Differential flatness has proven to be a very powerful concept for trajectory tracking control in any dynamic locomotion problem. The merit is that once a system is proved to be differentially flat and the flat outputs are determined, planning and control can be done in the flat output without worrying about the differential equations, as every flat output trajectory automatically satisfies the governing differential equations.

There is another reason for the advantage of the differential flatness approach is that in the event that a new trajectory has to be computed (due to a change in the parameters of the system or due to a change in the initial conditions), this can be done by substituting the new parameters and/or initial conditions in the appropriate formula, without having to solve a difficult non-linear programming problem again.

With the flatness property, it only requires to compute the flat outputs once, from which the control inputs can be found. This control strategy will greatly reduce the computation time by freeing it of some of the heavy and complicated optimization methods.

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In fact, the notion of flatness is motivated by two main ideas. First, it can be seen as a generalization of linear dynamical systems where all linear controllable dynamical systems are flat and static feedback linearizable [18,19]. Moreover, it provides another efficient algorithm to solve problems of input–output dynamic inversion of nonlinear systems. Indeed, systems that are differentially flat have several useful characteristics that can be exploited to generate effective control strategies for nonlinear systems.

However, the application of differential flatness to nonlinear control systems requires the knowledge of the flat outputs. Unfortunately, necessary and sufficient conditions do not exist to determine if a general system is differentially flat and there are no general algorithms to compute the flat outputs. Although we know that a system is flat there remains the problem of determining the flat outputs.

As already noted, since feedback linearizable dynamical systems are flat, some results in this direction are stated, such that controllable linear systems are differentially flat. It is also known that controllable affine dynamical systems with n states and $n-1$ inputs (codimension 1) are flat [20]. However a driftless system with n states and $n-1$ inputs is flat if and only if it is controllable [21]. A characterization of the so-called k-flatness with the Cartan–Kähler approach are given [22].

In fact, finding the expression of a flat output for general nonlinear systems appears to be much more difficult. The fundamental contribution of our paper is to present a new 0-flat normal form for a class of nonlinear dynamical systems whose outputs are flat. This method presents a new direction to solve the flatness problem. Sufficient geometrical conditions are given to transform the studied nonlinear systems into a 0-flat normal form and determine the flat outputs. As an extension of our results [23], where we have presented two new 0-flat normal forms of codimension 2 for a class of controllable nonlinear systems. We aim to control a five-link d.o.f biped robot to follow a specified trajectory using flatness based approach. We will characterize and give some conditions for a class of nonlinear dynamical systems under which the system exhibits differential flatness. The approach reveals a new design based on a 0-flat normal form, the flatness property of robotic model is derived and used in the development of the controller. Using the differential flatness concept, attained by normal 0-flat form, desired tracking trajectory is demonstrated, in contrast to the approach proposed in [24] where a partial feedback linearization based controller is used and the actuated joints response is linearized, it will be shown here that the proposed 0-flat normal form performs very well.

The paper is organized as follows. In the next section the biped robot prototype is described, then the dynamic model is presented, singling out the appropriate dynamic model for each phase of the walking cycle. Section 3 describes the classes of 0-flat systems study for affine dynamical systems, these classes will be characterized by their normal forms. We give the necessary and sufficient geometrical conditions for affine dynamical systems. Section 4, provides case study of a five-link biped robot. Finally simulation results are given in Section 5 to attest the efficiency of the proposed scheme and a conclusion ends the paper.

2. The rabbit prototype description

Rabbit is a prototype robot [25] (see Fig. 1) presenting a five-link under-actuated biped robot with seven degrees of freedom and 4 actuators, namely only the femurs and the tibias are actuated. Rabbit is aimed to experiment walking as well as running gaits without feet nor elastic actuators, furthermore it enables easily transitions between gaits. By means of a guidance device, RABBIT walks in a circular path (see Fig. 2). More technical details about the testbed can be found in [25].

Fig. 1. The prototype testbed.

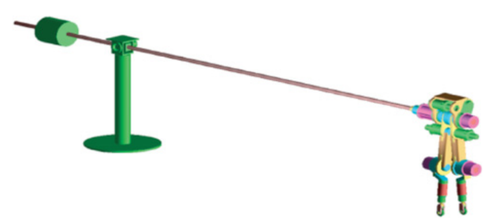

Fig. 2. The guidance device.

2.1. Dynamic model

The Lagrange formalism [26] enables the mathematical model describing the biped moving in the sagittal plane as follows:

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + G(q) = Su \quad (1)$$

where $M(q) \in \mathbb{R}^{7\times 7}$ is the inertia matrix, $N(q, \dot{q}) \in \mathbb{R}^{7\times 7}$ contains the centrifugal and Coriolis forces terms, $G(q) \in \mathbb{R}^{7}$ is the vector of gravitational forces, $u = [u_1\ u_2\ u_3\ u_4]^T \in \mathbb{R}^4$ is the vector of control inputs, S is a torque distribution matrix, $q = [q_{31}\ q_{41}\ q_{32}\ q_{42}\ q_1\ x\ y]^T \in \mathbb{R}^7$ is the vector of generalized coordinates (see Fig. 3). It is assumed that the walking movements take place in the sagittal plane, and on a horizontal surface without obstacles.

It is generally accepted that a walking cycle includes two sequential phases of motion: single-support (SS), with one foot grounded and double-support (DS), both feet grounded. Accordingly, the dynamic model is composed of two sets of equations, each corresponding to a phase of motion. In the case-study of this note, we have 7 d.o.f in the phase of flight (both feet in the air during running gaits), five degrees of freedom in the SS phase and three degrees in the DS phase.