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Dimensional synthesis of kinematically redundant serial manipulators for cluttered environments

Ekta Singla, Suryamani Tripathi, Venkataramani Rakesh, Bhaskar Dasgupta *

Department of Mechanical Engineering, Indian Institute of Technology, Kanpur 208 016, India

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ABSTRACT

A systematic procedure for synthesizing kinematically redundant serial manipulators is proposed in this paper. For a given cluttered workcell, the task space locations (TSL's) for the desired manipulator are prescribed. The synthesis is performed with the objective of reachability of the manipulator at specified TSL's, while avoiding obstacles. The problem is formulated as a constrained optimization problem, minimizing the positional error and simultaneously avoiding any collision of the manipulator with either the obstacles or within its links. The technique used to solve the resulting constrained optimization problem is the classical Augmented Lagrangian Method. The paper presents a discussion on the past works in this field. It is observed that the presented literature is confined to special cases only while the proposed method involves full generality of the synthesis problem. The availability of such an algorithm working for full generality is important, particularly for highly constrained environments. The efficiency of the proposed approach to synthesize the desired redundant manipulators is exhibited through diverse cases. The resulting synthesized manipulators are further checked for the possibility of feasible paths between TSL's. An outline of the development of a redundant manipulator, synthesized through the presented approach, is also included in this paper.

1. Introduction

Redundancy in robot manipulators is introduced to gain dexterity and/or autonomy. Different kinds of redundancy that may be present in a manipulator are *kinematic redundancy*, *sensor redundancy*, *actuation redundancy* and *force redundancy*. This paper focusses on kinematically redundant serial manipulators. The implementation of redundant manipulators is attractive due to extra capabilities exhibited by these mechanisms. Using the feature of kinematic redundancy, we achieve enhancements in different properties of manipulators like reachability, dexterity and workspace volume. It is also utilized for obstacle avoidance, singularity avoidance and fault tolerance. The presence of kinematic redundancy gives infinite solutions to the inverse kinematics problem and choosing a suitable solution out of these is non-trivial. A lot of research has been devoted to the quest for developing efficient procedures to solve the inverse kinematics problem of redundant manipulators [2–7].

Kinematic synthesis of a manipulator is to determine its design parameters satisfying certain kinematic requirements. Among these, the first one may be the ability of the manipulator to reach the specified locations. Other priorities down the line can be anything like obstacle avoidance, singularity avoidance, dexterity of the manipulator and several other objectives which affect the kinematic design of manipulators directly or indirectly to various extents. An important concept of reconfigurable manipulator system was floated by Lee and Lee [8] and an optimal synthesis procedure for reconfigurable manipulators was given by Paredis et al. [9,10]. The algorithm determines Denavit–Hartenberg (D–H) parameters for non-redundant manipulators within specified joint limits. As the approach is limited to 6-DOF manipulators, it cannot be utilized for redundant cases. Besides, the environments can include just parallelopiped-shaped obstacles, which is a relatively minor issue as normally any workspace can be approximated by such primitives. Many researchers [11–15], in the last decade, have worked on synthesis of spatial mechanisms using analytical and optimization approaches. Though their work illustrates the strategy of using optimization for dimensional synthesis problems, they deal with only a few DOF's. Our emphasis, in this article, is on higher DOF's and that too for cluttered environments. The published work that comes closest to our theme is the design of hyper-redundant manipulators for enclosed spaces by Wunderlich [16], who justify the importance of manipulator design for specific tasks on the need for executing a limited number of tasks in an inaccessible location an enormously large number of times, as encountered in the automobile industry. Our motivation for the current work comes from a similar situation in the nuclear industry, where well-defined routine maintenance

* Corresponding author. Tel.: +91 512 2597995; fax: +91 512 2597408.
E-mail address: dasgupta@iitk.ac.in (B. Dasgupta).

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Nomenclature

$\mathbf{x}$	Vector of design variables
n	Number of links
N	Number of task space locations (TSL's)
$f(\mathbf{x})$	Objective function to be minimized
$\mathbf{g}(\mathbf{x}), \mathbf{h}(\mathbf{x})$	Vectors of inequality and equality constraint
$F(\mathbf{x})$	Augmented Lagrangian function
μ_i, λ_j	Lagrange multipliers corresponding to inequality and equality constraints, for $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$
I	Number of inequality constraints
J	Number of equality constraints
P_{error}	Total cumulative error for all the N TSL's
$\mathbf{r}_{td}, \mathbf{r}_{ta}$	Desired and actual position vectors of t-th TSL, for $t = 1, 2, \ldots, N$
$[a_{s-1}, \alpha_{s-1}, d_s, \theta_s]$	D–H parameters [1] corresponding to s-th link of a manipulator, for $s = 1, 2, \ldots, n$.
$distP(\mathbf{x})$	Function defined for collision detection.

work is needed to be carried out at much more complicated workcells. Wunderlich [16] essentially presents a synthesis of feasible designs of manipulators in planar workcells for a single task, for which they select the most difficult of the tasks expected of the manipulator. For a planar problem, they are able to decide on an initial trial set of link-lengths easily by visual inspection and then permutations of the link-lengths are tried for an improvement. The present authors' research group had also made an attempt in that direction [17], in which a sufficiently large total length was iteratively distributed in a *quantized* manner among a large number of link lengths to arrive at a feasible design. It was noticed that for really complicated workcells (contrary to a single enclosed space) with several task locations at far-flung zones, such an approach becomes prohibitively costly. For spatial (3-d) workcells, the situation is worse with larger number of unknown D–H parameters and no initial feasible solutions easily available. Such scenarios encountered in actual nuclear plants motivate our current study of this challenging problem.

Section 2 briefly outlines the tools used in the present work. Problem formulation, constraint handling and solution methodology are discussed in Section 3. Section 4 includes a set of examples to verify and evaluate the proposed approach. Besides, the section briefly presents the development of a serial redundant manipulator, synthesized using our method. The work is concluded in the last section with a summary and discussion on future possibilities.

2. Computational tools

In this section, the optimization technique, the augmented Lagrangian method, is briefly reviewed. This technique has been chosen based on its robustness and ability to maneuver through the infeasible zones of the design space quite comfortably. This is important for our problem as the feasible regions are sparsely distributed and it is difficult to access them without crossing infeasible ones. Introductions to the Proximity Query Package (PQP) and Probabilistic Roadmap path planner (PRM), which have been utilized for the purpose of obstacle avoidance and path verification respectively, are also included in this section.

2.1. Augmented Lagrangian method

This method of constrained optimization was conceptualized by Powell, Hestenes and Rockafellar [18–20]. A pure penalty function method penalizes an objective function in order to discourage constraint violation, typically with a large penalty parameter. This results in steeply curved functions and in turn causes a poor rate of convergence, since in such situations the Hessian of the Lagrangian becomes ill-conditioned [21,22]. On the other hand, a purely dual method would require the function to be convex, which is impractical for the initial iterations. In the augmented Lagrangian method, the Lagrangian is combined with a modest penalty term, only to render the 'augmented Lagrangian' convex with respect to the primal variables. A value of the penalty parameter sufficient for this purpose clearly depends on the eigenvalue structure of the Hessian at a point in the solution space. Since the eigenstructure analysis at every iteration would incur an enormously wasteful computational exercise, we conduct it only at the starting solution. Whatever happens to be the least penalty parameter value for convexity at the starting point, we use twice that value for all iterations so as to ensure that the penalty term always be sufficient to render the augmented Lagrangian convex. This turns out to be quite adequate, since iterations by and large proceed *towards* convexity. The augmented Lagrangian is minimized to get the optimal solution of the primal problem. In the process, the Lagrange multipliers are updated in each iteration and the penalty parameters need not be very large. This avoids the sharp modifications in the function contours and thus helps in smooth and gradual convergence.

To briefly explain the approach, we suppose that $\mathbf{x}$ is the vector of design variables and $f(\mathbf{x})$ is the objective function to be minimized subject to the inequality constraints

$$g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \ldots, I;$$

and equality constraints

$$h_j(\mathbf{x}) = 0, \quad j = 1, 2, \ldots, J;$$

where I is the number of inequality constraints and J is the number of equality constraints. The augmented Lagrangian function to be optimized with equality and inequality constraints is given as

$$F(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^{I} \mu_i g_i(\mathbf{x}) + \frac{1}{2} r_1 \sum_{i=1}^{I} [\max(0, g_i(\mathbf{x}))]^2 + \sum_{j=1}^{J} \lambda_j h_j(\mathbf{x}) + \frac{1}{2} r \sum_{j=1}^{J} [h_j(\mathbf{x})]^2 \quad (1)$$

where μ_i and λ_j are the Lagrange multipliers corresponding to the i-th and j-th inequality and equality constraints, respectively; whereas r and r_1 are external penalty parameters.

For the optimal solution of this unconstrained function, the Polak–Ribiere formula, a version of the conjugate gradient method, has been utilized. In each iteration of this method a successive conjugate direction is determined as

$$\mathbf{d}_k = -(\nabla F)_k + \beta_{k-1} \mathbf{d}_{k-1}, \quad (2)$$

which works as the search direction for that iteration. $(\nabla F)_k$ represents the gradient of the augmented Lagrangian and β_{k-1} is given by the Polak–Ribiere formula as

$$\beta_{k-1} = \frac{(\nabla F)_k^T ((\nabla F)_k - (\nabla F)_{k-1})}{(\nabla F)_{k-1}^T (\nabla F)_{k-1}}.$$

The line search in the direction $\mathbf{d}_k$ is performed by the golden section method for updating $\mathbf{x}$ as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \gamma_k \mathbf{d}_k. \quad (3)$$

This iterative process terminates when the gradient $(\nabla F)_{k+1}$ becomes close to zero. Convergence of this iterative process completes one round of the inner loop and at this solution, the