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Finite-time tracking control for robot manipulators with actuator saturation

Yuxin Su [*,1], Jan Swevers

Department of Mechanical Engineering, Division PMA, KU Leuven, Celestijnenlaan 300B, B-3001 Leuven-Heverlee, Belgium

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ABSTRACT

This paper addresses the finite-time tracking of robot manipulators in the presence of actuator saturation. The commonly-used proportional-derivative (PD) plus dynamics compensation (PD+) scheme is extended by replacing the linear errors in the PD+ scheme with saturated non-smooth but continuous exponential-like ones. Advantages of the proposed controller include semi-global finite-time tracking stability featuring faster transient and high-precision performances and the ability to ensure that actuator constraints are not violated. This is accomplished by selecting control gains *a priori*, removing the possibility of actuator failure due to excessive torque input levels. Lyapunov's direct method and finite-time stability are employed to prove semi-global finite-time tracking. Simulations performed on a three degree-of-freedom (DOF) manipulator are provided to illustrate the effectiveness and the improved performance of the formulated algorithm.

1. Introduction

One of the basic functionalities of robot manipulators is trajectory tracking. Several control schemes to implement tracking tasks of robot manipulators can be found in the literature [1–4]. While these control schemes are elegant and intuitively appealing, there is an implicit assumption in the development of these schemes that the robot system actuators are able to provide any requested torque. It is known that if the controller requests more torque than the actuators can supply, degraded or unpredictable motion and thermal or mechanical failure may result [5–7].

Recognizing these difficulties, several solutions that take into account actuator constraints during robot trajectory tracking have been proposed. For example, Loria and Nijmeijer [8] first addressed the semi-global asymptotic tracking of Euler-Lagrange systems with saturated position feedback PD plus feedforward dynamics (PD+) scheme. Aguinga-Ruiz et al. [9] proposed state feedback saturated PD+ control for global asymptotic stability with a restrictive and limitative condition that the inherent damping friction coefficient on each joints is larger than the upper boundedness of the desired trajectories. Lefeber and Nijmeijer [10] combined a bounded regulation controller with a local asymptotically stable tracking controller and achieved global asymptotic tracking. Dixon et al. [11] formulated a saturated output feedback PD+ scheme and saturated adaptive control and obtained semi-global asymptotically tracking. Moreno-Valenzuela et al. [12] incorporated gains scaling of the argument of the hyperbolic tangents into the saturated PD+ control and showed the local exponential stability. Recently, Su and Zheng [13] developed a simple decentralized saturated repetitive learning controller for semi-global asymptotic tracking of robot manipulators.

However, these bounded controllers only achieve asymptotic tracking stability implying that the system trajectories converge to the equilibrium as time goes to infinity. It is known that finite-time stabilization of dynamical systems may give rise to fast transient and high-precision performances besides finite-time convergence to the equilibrium [14–18].

This observation is supported by a review of literature which yields different approaches to address finite-time tracking of robot manipulators, which implies that the tracking errors become and remain zero within a finite time. Specifically, Man et al. [19] presented a robust terminal sliding mode (TSM) control for rigid robotic manipulators. Tang [20] developed an improved TSM control to implement global finite-time tracking. Feng et al. [21] addressed the singularity problem of the TSM control. Yu et al. [22] proposed a continuous TSM controller by using a new form of terminal sliding mode. Jin et al. [23] presented a practical nonsingular TSM tracking control by using time-delay estimation technique. Zhao et al. [24] developed a new terminal sliding mode control approach for robotic manipulators based on finite-time stability theory and differential inequality principle. To cope with model uncertainty, Barambones and Etxebarria [25] incorporated adaptive control into TSM for high precision tracking of uncertain

* Corresponding author.
E-mail addresses: yxsu@mail.xidian.edu.cn, yuxin.su@mech.kuleuven.be (Y. Su), jan.swevers@mech.kuleuven.be (J. Swevers).

[1] He is on leave from the School of Electro-Mechanical Engineering, Xidian University, Xi'an, China.

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robot manipulators. Parra-Vega and Hirzinger [26] proposed a dynamic sliding controller to implement the perfect tracking in finite time. Liu and Zhang [27] proposed a neural network-based robust finite-time controller with the consideration of actuator dynamics. On the other hand, Su [28] and Su and Zheng [29] formulated an alternative continuous finite-time tracking control for robot manipulators by using geometric homogeneity technique. Although these finite-time tracking schemes give a faster transient and higher precision, the desired favourable performances relies heavily on the assumption that the robot system actuators are able to provide any requested torque.

This paper proposes a saturated finite-time tracking controller for robot manipulators with the consideration of actuator constraints. This is accomplished by replacing the linear errors in the commonly used PD+ with saturated non-smooth exponential-like ones. Lyapunov stability and finite-time stability theory are employed to prove semi-global finite-time tracking stability. The practical implications are that the actuators can be appropriately sized without an ad hoc saturation scheme to protect the actuator. Simulations are presented to demonstrate the improved performance of the proposed approach. Throughout this paper, the norm of a vector $x \in \Re^n$ is defined as $\|x\| = \sqrt{x^T x}$ and that of a matrix A is the corresponding induced norm $\|A\| = \sqrt{(A^T A)_M}$, where A_M denotes the maximum eigenvalue of the matrix A.

2. Preliminaries

2.1. Robot model and properties

The dynamics of a rigid revolute joint robot manipulator can adequately be described as [4]

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+D\dot{q}+g(q)=\tau, \tag{1}$$

where $q, \dot{q}$ and $\ddot{q} \in \Re^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \Re^{n\times n}$ is the symmetric inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ denotes the centrifugal-Coriolis matrix, $D \in \Re^{n\times n}$ represents the matrix composed of damping friction coefficients, $g(q) \in \Re^n$ denotes the influence of gravity, and $\tau \in \Re^n$ is the input torque. The following properties of robot manipulator dynamics (1) have been established [4].

Property 1. The inertia matrix $M(q)$ is symmetric positive definite and satisfies the following inequality:

$$m_1\|\zeta\|^2 \le \zeta^T M(q)\zeta \le m_2\|\zeta\|^2, \quad \forall q,\zeta \in \Re^n, \tag{2}$$

where m_1 and m_2 are known positive constants.

Property 2. There are positive constants d_M and c_2 such that

$$\|D\| \le d_M, \quad \|C(q,\dot{q})\| \le c_2\|\dot{q}\|, \ \ \forall q,\dot{q} \in \Re^n. \tag{3}$$

Property 3. The matrix $\dot{M}(q)-2C(q,\dot{q})$ is skew-symmetric, i.e.

$$\zeta^T(\dot{M}(q)-2C(q,\dot{q}))\zeta = 0, \quad \forall q,\dot{q},\zeta \in \Re^n, \tag{4}$$

which implies that

$$\dot{M}(q) = C(q,\dot{q})+C^T(q,\dot{q}), \quad \forall q,\dot{q},\zeta \in \Re^n. \tag{5}$$

Property 4. The vector $g(q)$ is bounded for all $q \in \Re^n$, i.e., there exist finite constants $\kappa_{gi} \ge 0$ such that

$$\sup_{q\in\Re^n} |g_i(q)| \le \kappa_{gi}, \quad i=1,\cdots,n, \tag{6}$$

where $g_i(q)$ denotes the ith component of $g(q)$.

2.2. Fundamental facts

Some concepts of finite-time stability of nonlinear systems are reviewed following the approach of Bhat and Bernstein [14] and Hong et al. [15]. Consider the system

$$\dot{x} = f(x), \quad f(0)=0, \quad x \in \Re^n \tag{7}$$

with $f: U_0 \to \Re^n$ continuous on an open neighborhood U_0 of the origin. Suppose that system (7) possesses unique solutions in forward time for all initial conditions. The equilibrium $x=0$ of system (7) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin. The finite-time convergence means the existence of a function $T: U\setminus\{0\} \to (0,\infty)$, such that, $\forall x_0 \in U \subset \Re^n$, the solution of (7) denoted by $s_t(x_0)$ with x_0 as the initial condition and $s_t(x_0) \in U\setminus\{0\}$ for $t \in [0, T(x_0))$, and $\lim_{t\to T(x_0)} s_t(x_0) = 0$ with $s_t(x_0)$ for $t > T(x_0)$. When $U=\Re^n$, we obtain the global finite-time stability.

A scalar function $V(x)$ is homogeneous of degree $\kappa \in \Re$ with $(r_1,\cdots,r_n)$, $r_i>0$, $i=1,\cdots,n$, if for any given $\varepsilon>0$,

$$V(\varepsilon^{r_1}x_1,\ldots,\varepsilon^{r_n}x_n) = \varepsilon^{\kappa}V(x), \quad \forall x \in \Re^n. \tag{8}$$

A continuous vector field $f(x)=[f_1(x),\cdots,f_n(x)]^T$ is homogeneous of degree $\kappa \in \Re$ with $r=(r_1,\cdots,r_n)$, if for any given $\varepsilon>0$,

$$f_i(\varepsilon^{r_1}x_1,\cdots,\varepsilon^{r_n}x_n) = \varepsilon^{\kappa+r_i}f_i(x), \ \ i=1,\cdots,n, \forall x \in \Re^n. \tag{9}$$

Some of the results on finite-time stability of a nonlinear system in [15] that will be used in this paper are summarized by the following two lemmas.

Lemma 1. *Consider the following system*

$$\dot{x} = f(x)+\hat{f}(x), \quad f(0)=0, \quad \hat{f}(0)=0, \quad x \in \Re^n, \tag{10}$$

where $f(x)$ is a continuous homogeneous vector field of degree $\kappa<0$ with respect to $(r_1,\cdots,r_n)$. Assume that $x=0$ is an asymptotically stable equilibrium of the system $\dot{x}=f(x)$. Then $x=0$ is a locally finite-time stable equilibrium of the system (10) if

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_i(\varepsilon^{r_1}x_1,\cdots,\varepsilon^{r_n}x_n)}{x^{\kappa+r_i}} = 0, \quad i=1,\cdots,n, \forall x \ne 0. \tag{11}$$

Lemma 2. *Semi-global asymptotic stability and local finite-time stability imply semi-global finite-time stability.*

3. Control design

3.1. Control objective

Let $q_d \in \Re^n$ be any reference trajectory for system (1) that is continuously differentiable up to its second derivative such that

$$\|\dot{q}_d\| \le V_M, \quad \|\ddot{q}_d\| \le A_M. \tag{12}$$

We also assume that each joint actuator has a maximum torque $\tau_{i,\max}$ that satisfies

$$\tau_{i,\max} > T_{i,M} \tag{13}$$

with

$$T_{i,M} = m_2A_M + c_2V_M^2 + d_MV_M + \kappa_{gi} \tag{14}$$

The output tracking errors are defined as

$$e = q-q_d, \dot{e} = \dot{q}-\dot{q}_d \tag{15}$$

Our objective is to design a saturated control input $\tau(t)$ such that $\lim_{t\to T(x_0)} e(t), \dot{e}(t) = 0$ while respecting the actuator constraints

$$|\tau_i| \le \tau_{i,\max}, \tag{16}$$