Robotics and Computer-Integrated Manufacturing 30 (2014) 201–205

Contents lists available at ScienceDirect

Robotics and Computer-Integrated Manufacturing

journal homepage: www.elsevier.com/locate/rcim

Numerical solution for designing telescopic manipulators with prescribed workspace points

Ericka Madrid [a], Marco Ceccarelli [b,*]

[a] Pontificial Catholic University of Perù, Perù
[b] University of Cassino and South Latium, Italy

ARTICLE INFO

Article history:
Received 16 May 2013
Received in revised form
16 September 2013
Accepted 24 September 2013
Available online 25 October 2013

Keywords:
Robotic manipulators
Workspace
Design
Numerical synthesis

ABSTRACT

In this paper a numerical solution is proposed for designing telescopic manipulators when workspace is prescribed through few suitable points. An algorithm is outlined by using an algebraic formulation for the workspace boundary and numerical solution is worked out by using a Newton–Raphson technique to solve a proper design problem. Numerical examples are reported to discuss computational efforts and solution characteristics.

1. Introduction

Telescopic manipulator arms are widely used in robots since a prismatic pair as last joint in the chain gives additional workspace capability.

Thus, workspace analysis has addressed and still addresses great attention in works in a large literature for developing procedures for fairly simple determination of the workspace volume and its characteristics as well as for rationales in classification of workspace possibilities. There is a very rich literature dealing with modeling, formulating, and computing workspace characteristics mainly with numerical procedures.

Dimensional design of manipulators is conveniently formulated by using workspace characteristics since manipulator reach ranges are recognized fundamental both for operation characterization and design purposes.

Thus, in general algorithms for workspace determination have been used also in design procedures for reiterative analysis or for inverted formulation. Analysis algorithms can be based on Direct Kinematics formulation by using matrix approach, Jacobian evaluation, and singularities identification in general purposes like those that are presented for example in [12,23,26,9,14,13,16,18,19,5,6,15,2,3].

Inversion of workspace formulation can be used for specific manipulator architectures with the aim to investigate parameters effects and to deduce specific design algorithms like, for example, for 3R manipulators in [20,10,24,11,4,1,17,25].

In this paper the attention is focused on the specific architecture of telescopic manipulators with one prismatic joint at the end of the open kinematic chain having two revolute pairs. Practical applications of telescopic manipulator architecture can be found not only in robotic arms, but even in machinery like for example excavators, firemen stairs, and cleaning arms. Telescopic manipulator arms are widely used in robots since a prismatic pair as last joint in the chain gives additional workspace capability, but no great specific attention has been directed to workspace analysis and design of this robot structure.

The design problem of telescopic arms has been attached specifically in few cases. Design equations have been formulated in [22] by using expressions for an equivalent screw triangle from the Screw Theory. Models with Denavit–Hartenberg parameters have been used in Lee and Mavroidis, 2008 in a design algorithm with polynomial elimination techniques. Analytical expression via dual quaternions has been formulated in [18] for serial chains with two prismatic joints.

This paper is an attempt to extend the procedure in [4,6] to the case of manipulator chains with prismatic pairs. In particular the formulation for the workspace of telescopic arms in ([5]) has been expressed explicitly for design purposes. Preliminary results were presented in [7,8]. In this paper a further development for design aims is proposed to discuss numerical solution of the design formulation and characteristics of design outputs as related to workspace prescriptions.

* Corresponding author. Tel.: +39 776 299 3663.
E-mail addresses: madrid.ep@pucp.edu.pe (E. Madrid), ceccarelli@unicas.it (M. Ceccarelli).

0736-5845/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.rcim.2013.09.013

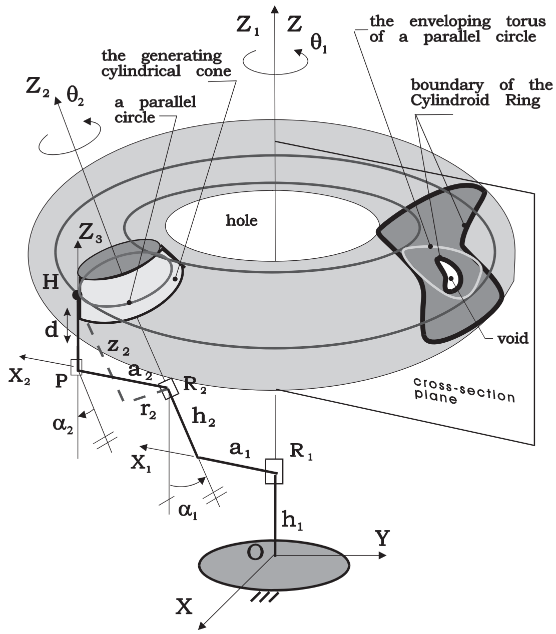

Fig. 1. Scheme for design parameters and workspace of telescopic manipulator arms.

2. Telescopic chain and design parameters

Fig. 1 shows a scheme for the kinematic chain of a telescopic manipulator together with a geometric illustration of the workspace manifold and its generation. In particular, in Fig. 1 a world frame OXYZ has been assumed as fixed with the manipulator base with the Z axis coincident with the first joint axis and X axis as the reference axis for the first joint coordinates. Moving frames $O_i X_i Y_i Z_i$ (i=1,2,3) have been fixed on the links of the chain by assuming O_i in the center of a joint, Z_i as coinciding with the joint axis, and X_i laying on the common normal between two consecutive joint axes. X_3 has been assumed to be parallel to X_2.

The design HD parameters are the link lengths a_1 and a_2, the link offsets h_1 and h_2, the twist angles α_1 and α_2. However h_1 is considered superfluous since it does not affect the workspace but only shifts it up or down. The stroke excursion for d is the design variable of the telescopic motion with the minimum and maximum values d_{min} and d_{max}, respectively.

The time variable kinematic parameters are the joint angles θ_1 and θ_2 of the revolute pairs R_1 and R_2 and the joint stroke d is the variable for the sliding pair P. Each joint coordinate starts from a line which is parallel to X-axis of previous link. The joint angles θ_1 and θ_2 are not considered as design variables since usually they vary for a full rotation and do not affect the workspace capability.

3. A formulation for workspace determination

The workspace is generated by a reference point H on the extremity of the telescopic chain when H is moved to reach all the possible positions because of the movements of the joints. Workspace of a telescopic manipulators can be characterized by looking at the loci which are generated by H because of successive movements of the joints starting from the last up to first, which is fixed to the manipulator base, Fig. 1. Thus, movements of the sliding joint P generate a straight line segment, which is limited by d_{min} and d_{max}. Then, the second revolute pair R_2 of the chain performs a full rotation of the straight line segment and generates a hyperboloid. Indeed, depending on the orientation of the straight-line segment with respect to the revolute joint axis Z_2 we may have a cylinder, a cone or generally a hyperboloid. Finally, a full revolution of the first revolute joint R_1 generates a solid of revolution with a general cross-section shape shown in Fig. 1. The workspace $W(H)$ shows a hollow bulk shape and its cross-section is characterized by straight lines with possible cusps and two circular contours on the top and bottom.

The described procedure for workspace boundary generation can be synthesized in computing the workspace boundary as an envelope of the tori that are traced by mobility in R_1 and R_2 for the points of the segment due to P motion. In particular, each point of the straight line segment is individuated through a value of the stroke d and the corresponding torus can be written by assuming h_1=0, with the hypothesis sin $\alpha_1 \neq 0$ and with respect to $OXYZ$, by using the radial reach $r=(x^2+y^2)^{1/2}$ and the axial reach z in the form

$$(r^2+z^2-A)^2+(Cz+D)^2+B=0 \tag{1}$$

in which the so-called structural coefficients are expressed as

$$\begin{aligned} A &= a_1^2+r_2^2+(z_2+h_2)^2 \\ B &= -4a_1^2 r_2^2 \\ C &= 2\frac{a_1}{s\,\alpha_1} \\ D &= -2a_1(z_2+h_2)\frac{c\alpha_1}{s\alpha_1} \end{aligned} \tag{2}$$

where r_2 and z_2 are the radial and axial reaches with respect to $O_2X_2Y_2Z_2$. The distances r_2 and z_2 can be given from the geometry of the chain as

$$\begin{aligned} r_2 &= \sqrt{a_2^2+d^2\sin^2\alpha_2} \\ z_2 &= d\cos\alpha_2 \end{aligned} \tag{3}$$

where the independent variable is the stroke parameter d.

Eqs. (1) to (3) can be used to determine the workspace volume for a given telescopic arm by scanning the stroke interval from a minimum value d_{min} to a maximum value d_{max}.

Indeed, a torus is traced by using Eq.(1) to determine r by assuming z. This is obtained by scanning z within its range from minimum to maximum values, which can be calculated by using the model of Fig. 1 from

$$z = r_2\sin\theta_2\sin\alpha_1+(h_2+d\cos\alpha_2)\cos\alpha_1+h_1 \tag{4}$$

Fig. 2 shows a numerical example of the workspace determination for a given telescopic manipulator in an excavator machine by using Eqs. (1) to (4). Fig. 2 shows that the workspace boundary is composed of two different loci: envelope segments and toroidal surfaces. The envelope segments are located in the lateral sides of the cross-section representation, and two toroidal surfaces are the top and bottom covers, respectively. The envelope segments can be determined by a formulation by using the Theory of Envelopes to obtain a more efficient computational algorithm for workspace determination, Figs. 1 and 2.

Thus, it is known that an analytical expression of the envelope can be obtained from Eq. (1) and its derivative with respect to the envelope parameter, which is the stroke d. Thus, differentiating Eq. (1) with respect to d we obtain

$$-(r^2+z^2-A)A'+(Cz+D)D'+B'=0 \tag{5}$$

where the symbol $'$ indicates d-derivative operator.

By using Eqs. (2) and (3) expressions for derivatives of the structural coefficients can be obtained in the form

$$\begin{aligned} A' &= 2(d+h_2\cos\alpha_2) \\ B' &= -4a_1^2 d\sin^2\alpha_2 \\ D' &= -2a_1\cos\alpha_2\frac{\cos\alpha_1}{\sin\alpha_1} \end{aligned} \tag{6}$$