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A comparative study of planar 3-RRR and 4-RRR mechanisms with joint clearances

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ABSTRACT

This paper presents a comparative study of the fully actuated 3-RRR mechanism and the redundantly actuated 4-RRR mechanism. The joint clearances are taken into consideration in both kinematic and dynamic analysis of the two mechanisms. In the kinematic analysis, the joint clearance is treated as a mass-less virtual link, and the error transfer equations of these mechanisms are obtained by the first derivation of the constraint equations. In the dynamic analysis, the joint clearance is fully described and the dynamic equations of the two mechanisms are derived using the Newton–Euler method. The dynamic equations are solved by using the two-step Bathe integration method. The results indicate that the 4-RRR mechanism shows better performances in both kinematics and dynamics when compared with the 3-RRR mechanism.

1. Introduction

Parallel robots are increasingly used for fast manipulation and accurate positioning, in which 3-RRR parallel mechanism is practical since it has high stiffness and large workspace [1–3]. It performs planar three degrees of freedoms, two translations along the *x*- and *y*-axes, and a rotation around the *z*-axis. 3-RRR parallel mechanism is fully actuated and studied by many researchers, it has larger workspace than the other type planar 3DOF parallel mechanism such as the 3-PRR mechanism. Some limitations in the functional workspace are often encountered, the most salient drawback is that there are many singular configurations within the workspace. Actuation redundancy [4,5] is a feasible way to reduce singular configurations among other methods, a planar 3DOF 4-RRR parallel mechanism can realize the same motion as the 3-RRR one.

Joint clearance is unavoidable due to manufacturing tolerances, assembling, wear and material deformation, leading to important deviations between the ideal behavior and real outcome of the mechanism [6]. The joint clearances also cause vibration, noise and wear, decreasing the service life or even leading to failure of the mechanism [7–9]. In the traditional analysis, the joints are assumed to be ideal and its effects are ignored. Due to the increasing requirements for high precision and high speed of the mechanism, it is necessary to analyze the multibody system with joint clearances.

In the kinematics analysis, Zhu [10] analyzed the uncertainty of both planar and spatial robots with joint clearances. Chen [11] proposed a unified approach to the accuracy analysis of planar parallel manipulators both with input uncertainties and joint clearance. Flores [12–14] studied the dynamics of multibody systems with spatial revolute joints and the spherical joints, in which the lubrication are taken into consideration. Lopes [15] developed a mathematical framework for contact detection between quadric and superquadric surfaces. Machado [16] and Alves [17] studied the influence of contact force models on the dynamic response of multibody systems. Considering the link flexibility, Tian [18] modeled the elastohydrodynamic lubricated cylindrical joints for rigid-flexible multibody dynamics. A Kriging model [19] is developed by Zhang to approximate the dynamics of mechanical systems with revolute joint clearances. Varedi [20] finished the optimal design of a planar slider-crank mechanism with a joint clearance. For specific cases, Zhao [21] analyzed the dynamics of space robot manipulator with joint clearance, while Erkaya [22] investigate the effects of joint clearance on welding robot manipulators. In the experimental aspect, Koshy [23] and Erkaya [24,25] did experimental studies to verify their theoretical analysis.

The kinematics and dynamics of 3-RRR and 4-RRR parallel mechanisms have been investigated by some researchers [26], however, only few researchers take the joint clearances into consideration. Zhang [1,2] studied the dynamics of 3-RRR parallel

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mechanism with multiply joint clearances. In Muller's work [4,5], the internal preload control of redundantly actuated parallel manipulators is applied to avoid backlashes in actuators, in which the 4-RRR mechanism is illustrated. Furthermore, there is no comparison study on the kinematics and dynamics of the two types of mechanisms both with joint clearances.

It is notable that most published literatures considering the joint clearances are focused on the 1DOF mechanism, very few researchers studied the dynamics of the multi DOFs mechanism with multi joint clearances, not to mention the redundant mechanism. So the main purpose of this paper is to show the different performances of the redundant and non-redundant mechanisms with multi joint clearances. The fully actuated 3-RRR mechanism and the redundantly actuated 4-RRR mechanism are used as illustrations. In the kinematic analysis, the joint clearance is treated as a mass-less virtual link. Based on the first derivation of the constraint equations, the error transfer equations of the two type of parallel mechanism are obtained. In the dynamic analysis, the joint clearance is modeled in details, while the dynamic equations of the two mechanisms are derived using the Newton–Euler method. After solving the motion equations of each mechanism using Bathe integration method, the simulation results are compared and discussed. This paper is helpful for the readers to understand the advantages and drawbacks of these two types of mechanisms.

2. Kinematics of the mechanism with joint clearances

The 3-RRR and 4-RRR mechanisms are symmetric, with all the parts are treated as rigid, thus the fixed and moving platforms of the two mechanisms are equilateral triangle/square. The global coordinate system XOY is fixed at the center of symmetry of the fixed platform, while the local coordinate systems are attached on the center of mass of each moving body, as shown in Fig. 1. Joints Ai are attached to the actuators, while joint Bi and Ci are revolute joints with clearance. At the active joint, the driving links are bolded with the axis of the actuators, the backlash of the motor and reducer are not considered in the analysis, the inputs are taken as ideal. In the 3-RRR mechanism, 6 joint clearances are taken into consideration, while in the 4-RRR mechanism, 8 joint clearances are taken into consideration.

Let l_1 and l_2 denote the length of active link *AB* and passive link *BC*, while let l_3 and l_4 denote the radius of the circumscribed circle of the moving and fixed platform. The reference point coordinates are used to formulate the system [27], thus each body can be described by three coordinates

$$\mathbf{q}_i = \left[x_i,\ \ y_i,\ \ \theta_i\right]^T \tag{1}$$

where x_i and y_i are the positions of the center of mass of body *i* in the global coordinate system, θ_i is the angle between local and global coordinate systems. For 3-RRR, $i=1$–6, while for 4-RRR, $i=1$–8. Particularly, the coordinates of the moving platform for both mechanisms are expressed as $\boldsymbol{q}_o = [x_o,\ \ y_o,\ \ \theta_o]^T$.

2.1. Inverse kinematics of 3-RRR mechanism

In this step, it is assumed that all the geometrical dimensions of the mechanism are ideal, that is, there is no dimensional tolerance or joint clearance. For the first chain of the 3-RRR mechanism, there exist the following constraint equations

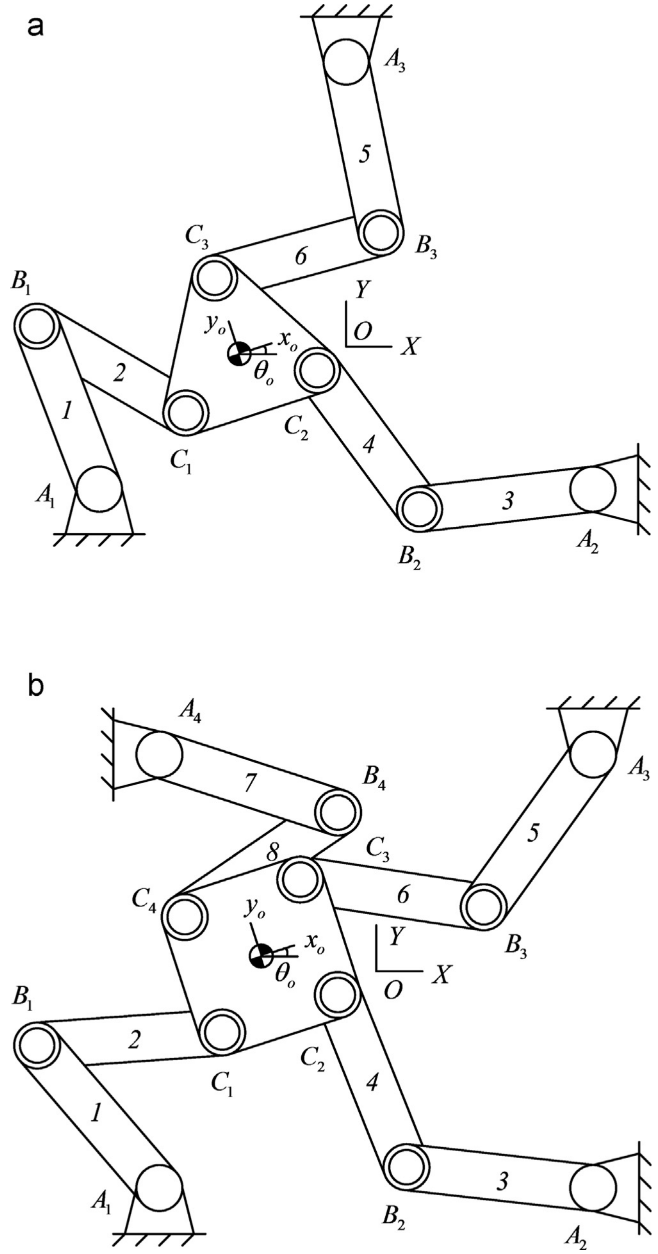

Fig. 1. Geometry of the two types of mechanisms with joint clearances. (a) 3-RRR mechanism. (b) 4-RRR mechanism.

$$\mathbf{\Phi}_1(\mathbf{q}) = \begin{bmatrix} x_1 - 0.5l_1\cos\theta_1 - x_{A1} \\ y_1 - 0.5l_1\sin\theta_1 - y_{A1} \\ x_2 - 0.5l_2\cos\theta_2 - (x_1 + 0.5l_1\cos\theta_1) \\ y_2 - 0.5l_2\sin\theta_2 - (y_1 + 0.5l_1\sin\theta_1) \\ x_0 + l_3\cos\left(\theta_0 + 7\pi/6\right) - (x_2 + 0.5l_2\cos\theta_2) \\ y_0 + l_3\sin\left(\theta_0 + 7\pi/6\right) - (y_2 + 0.5l_2\sin\theta_2) \end{bmatrix} = \mathbf{0} \tag{2}$$

In the kinematic analysis, the inputs and outputs are concerned. Eq. (2) can be rewritten as

$$\begin{cases} x_{A1} + l_1\cos\theta_1 + l_2\cos\theta_2 = x_0 + l_3\cos\left(\theta_0 + 7\pi/6\right) \equiv x_{C1} \\ y_{A1} + l_1\sin\theta_1 + l_2 sin\theta_2 = y_0 + l_3\sin\left(\theta_0 + 7\pi/6\right) \equiv y_{C2} \end{cases} \tag{3}$$

After some operations, the following equation can be obtained

$$w_1\cos\theta_1 + w_2\sin\theta_1 - w_3 = 0 \tag{4}$$