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The kinematic design of spatial, hybrid closed chains including planar parallelograms

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ABSTRACT

It is presented an integral approach for the kinematic design of spatial, hybrid closed chains which include planar parallelograms into their kinematic structure. It is based on a systematic application of recursive formulae intended for describing the evolution of screws through time. Due to the particular nature of the proposed approach, it can be closely related with Lie algebras and allows to overcome the lacking of group structure offered by a parallelogram when it is going to be considered as a component of a hybrid closed chain. Several application examples are presented in order to show the potential of the proposed approach.

1. Introduction

Early research on parallel manipulators was mainly focused on the analysis of motion types, number of degrees of freedom and structural synthesis. Since then, due to the immense number of potential applications offered by parallel manipulators, the variety of designs has not ceased to grow.

Among the large number of designs of parallel manipulators, there is an important and fundamental class which is characterized by a kinematic architecture that includes only a single closed chain. They are usually known as *parallel manipulators with two legs*, and they are basically *single closed chains*. On one hand, for normal closed chains — those with no link bifurcations where each link is only connected with two other links — most of the methods concerning with structural synthesis give acceptable results. On the other hand, when the closed chain under analysis contains link bifurcations — as the inclusion of a serially added parallelogram — problems arise. Such kinematic chains are usually known as *hybrid closed chains* (HCC for short) and they require a more elaborate formulation.

Since the introduction of parallel manipulators, a planar four-bar linkage — coupled by four revolute joints — with its opposite links of the same length (usually known as *parallelogram* or *Π joint*) has attracted the attention of several researchers[1] [1–18]. Thus, a suitable combination of a Π joint with other kinematic pairs leads to the design of HCCs. However, the Π joint lacks a group structure and therefore, defies a proper description by subgroups [7]. In order to overcome such a drawback for designing HCCs, and, consequently, for conducting the kinematic design of multi-loop parallel manipulators, some authors have just replaced a Π joint by a prismatic joint, *P*-type, by using a number of approaches: the Lie group algebraic properties of SE(3) [2,13], resorting to theory of linear transformations [17,18], considering special Plücker coordinates [9], based on the so-called G_F coordinates [11], and analyzing the rotational capabilities of the mechanisms [12], among others. While these authors did not give a theoretical or convenient reasoning behind replacing a Π joint by a *P*-joint, the analysis presented in this paper will readily explain the motion features associated with the Π joint. Indeed, the authors of Ref. [13] just briefly mention that for a small motion, the Π joint is equivalent to a prismatic pair. Moreover, some authors [2,3,17] also point out that the coupler link of a Π joint makes a translational circular motion with respect to its opposite link and preserves a constant orientation. Additionally, [4,8] went further and they report that the feasible motion of the coupler of a Π joint can be represented with a twist reciprocal to five restraining-screws. However, the authors of [4,8] do not mention what is the rationale behind the *finite motion* of the device under study. Hence the motivation to write this paper.

2. Motion type associated with the Π joint

The main problem that is addressed when a mechanical device or linkage is assembled from a set of links and joints is to determine

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[1] For that reason, this kinematic structure deserves its own symbol. To this end, and given the physical similarity with a parallelogram, we will adopt the symbol Π, which has been already proposed in [3].

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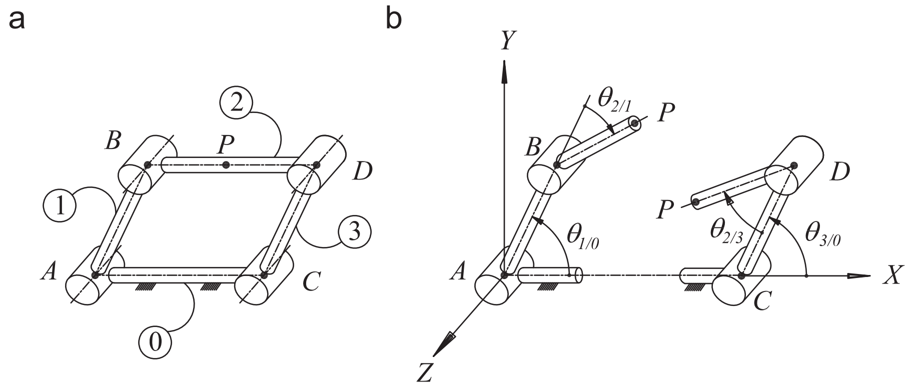

Fig. 1. A planar parallelogram: (a) schematic diagram and (b) corresponding serial legs.

the features associated with its resulting motion type. Thus, a natural question arises: Is the linkage mobile and hence a mechanism or is it a structure? With this idea in the mind, the objective of this section is to provide a detailed explanation concerning with the motion features associated with the Π joint, particularly with the motion of its coupler link.

It is well known that a planar 4R parallelogram, here called Π joint, is a single closed chain, see Fig. 1a. Let links 0 and 2 be the base and output (coupler) links of the Π joint. Thus, two RR-type serial legs, namely, *A*–*B* and *C*–*D*, constrain the motion of the coupler with respect to the fixed link. Hence, in order to know about the motion features of the coupler link with respect to the fixed link, we are going to assume that we just have the two serial chains appearing in Fig. 1b. For that reason, we do not even know, *a priori*, if such a pair of legs will be capable of producing a mobile linkage, where the coupler link will have a certain motion (not known yet) with respect to the fixed link.

It can be shown that the velocity state $\mathbf{V}_{AB}$, of body 2 with respect to body 0, which is associated with leg *A*–*B* is given by

$$\mathbf{V}_{AB} \equiv \begin{bmatrix} \boldsymbol{\omega}_{2/0} \\ (\mathbf{v}_{P/A})_{P\to A} \end{bmatrix} = \dot{\theta}_{1/0}\begin{bmatrix} \mathbf{k} \\ \mathbf{0} \end{bmatrix} - \dot{\theta}_{2/1}\begin{bmatrix} \mathbf{k} \\ \mathbf{r}_{B/A}\times\mathbf{k} \end{bmatrix} = \dot{\theta}_{1/0}\mathbf{S}_1 - \dot{\theta}_{2/1}\mathbf{S}_2, \tag{1}$$

$\mathbf{k}$ being a unit vector along *Z*-axis, $\mathbf{r}_{i/j}$ is a position vector going from point *j* to point *i*, $\mathbf{v}_{i/j}$ is the velocity vector of point *i* with respect to point *j*, $\omega_{m/n}$ is the angular velocity vector of link *m* with respect to link *n*, $\dot{\theta}_{k/l}$ is the first time derivative of angle $\theta_{k/l}$, and $\mathbf{S}_q$ denotes the *q*-th infinitesimal screw.

On the other hand, the velocity state $\mathbf{V}_{CD}$, of body 2 with respect to body 0, which is now associated with leg *C*–*D* is given by

$$\mathbf{V}_{CD} \equiv \begin{bmatrix} \boldsymbol{\omega}_{2/0} \\ (\mathbf{v}_{P/A})_{P\to A} \end{bmatrix} = \dot{\theta}_{3/0}\begin{bmatrix} \mathbf{k} \\ \mathbf{r}_{C/A}\times\mathbf{k} \end{bmatrix} - \dot{\theta}_{2/3}\begin{bmatrix} \mathbf{k} \\ \mathbf{r}_{D/A}\times\mathbf{k} \end{bmatrix} = \dot{\theta}_{3/0}\mathbf{S}_3 - \dot{\theta}_{2/3}\mathbf{S}_4, \tag{2}$$

where it should be noted that all the infinitesimal screws were referred to fixed point *A*.

On the other hand, from the geometry shown in Fig. 1, it is possible to formulate the following position vectors:

$$\mathbf{r}_{B/A} = L\cos\theta_{1/0}\mathbf{i} + L\sin\theta_{1/0}\mathbf{j}, \quad \mathbf{r}_{C/A} = H\mathbf{i}, \quad \mathbf{r}_{D/A} = H\mathbf{i} + L\cos\theta_{3/0}\mathbf{i} + L\sin\theta_{3/0}\mathbf{j}. \tag{3}$$

where *L* is the length of links 1 and 3, *H* is the distance between points *A* and *C*, $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along *X*- and *Y*-axes, respectively.

Thus, after performing the corresponding cross-products indicated in Eqs. (1) and (2), there are obtained the so-called *parametric form* of the infinitesimal screws:

$$\mathbf{S}_1 = \begin{bmatrix} 0\\0\\1\\0\\0\\0 \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} 0\\0\\1\\ L\sin\theta_{1/0} \\ -L\cos\theta_{1/0} \\ 0 \end{bmatrix}, \quad \mathbf{S}_3 = \begin{bmatrix} 0\\0\\1\\0\\-H\\0 \end{bmatrix}, \quad \mathbf{S}_4 = \begin{bmatrix} 0\\0\\1\\ L\sin\theta_{3/0} \\ -H-L\cos\theta_{3/0} \\ 0 \end{bmatrix}. \tag{4}$$

For such a set of screws, and, after a finite interval of time Δt, the new screws are going to be given by [19]

$$\mathbf{S}_1(t+\Delta t) = \mathbf{S}_1, \tag{5}$$

$$\begin{aligned} \mathbf{S}_2(t+\Delta t) = {} & \mathbf{S}_2 + \dot{\theta}_{1/0}[\mathbf{S}_1\ \mathbf{S}_2]\Delta t + \tfrac{1}{2}(\ddot{\theta}_{1/0}[\mathbf{S}_1\ \mathbf{S}_2] - \dot{\theta}_{1/0}\dot{\theta}_{2/1}[\mathbf{S}_1[\mathbf{S}_1\ \mathbf{S}_2])\Delta t^2 \\ & + \tfrac{1}{6}(\dddot{\theta}_{1/0}[\mathbf{S}_1\ \mathbf{S}_2] + 3\dot{\theta}_{1/0}\ddot{\theta}_{1/0}[\mathbf{S}_1[\mathbf{S}_1\ \mathbf{S}_2]] \\ & + \dot{\theta}^3_{1/0}[\mathbf{S}_1[\mathbf{S}_1[\mathbf{S}_1\ \mathbf{S}_2]]])\Delta t^3 \\ & + \tfrac{1}{24}(\dot{\theta}^4_{1/0}[\mathbf{S}_1[\mathbf{S}_1[\mathbf{S}_1[\mathbf{S}_1\ \mathbf{S}_2]]]] + \cdots)\Delta t^4 + \cdots, \end{aligned} \tag{6}$$

$$\mathbf{S}_3(t+\Delta t) = \mathbf{S}_3,$$

$$\begin{aligned} \mathbf{S}_4(t+\Delta t) = {} & \mathbf{S}_4 + \dot{\theta}_{3/0}[\mathbf{S}_3\ \mathbf{S}_4]\Delta t + \tfrac{1}{2}(\ddot{\theta}_{3/0}[\mathbf{S}_3\ \mathbf{S}_4] - \dot{\theta}_{3/0}\dot{\theta}_{2/1}[\mathbf{S}_3[\mathbf{S}_3\ \mathbf{S}_4])\Delta t^2 \\ & + \tfrac{1}{6}(\dddot{\theta}_{3/0}[\mathbf{S}_3\ \mathbf{S}_4] + 3\dot{\theta}_{3/0}\ddot{\theta}_{3/0}[\mathbf{S}_3[\mathbf{S}_3\ \mathbf{S}_4]] \\ & + \dot{\theta}^3_{3/0}[\mathbf{S}_3[\mathbf{S}_3[\mathbf{S}_3\ \mathbf{S}_4]]])\Delta t^3 \\ & + \tfrac{1}{24}(\dot{\theta}^4_{3/0}[\mathbf{S}_3[\mathbf{S}_3[\mathbf{S}_3[\mathbf{S}_3\ \mathbf{S}_4]]]] + \cdots)\Delta t^4 + \cdots \end{aligned} \tag{7}$$

equations from where the set of elements composing the Lie subalgebra or subspace associated with each leg can be directly obtained. It should be noted that $[\mathbf{S}_i\ \mathbf{S}_j]$, $[\mathbf{S}_i\ [\mathbf{S}_j\ \mathbf{S}_k]]$,..., are nothing but *nested Lie products* of screws.

On one hand, for leg *A*–*B* it can be obtained that[2]

$$\mathcal{S}_{AB} = \langle \mathbf{S}_1, \mathbf{S}_2, [\mathbf{S}_1\ \mathbf{S}_2] \rangle$$

[2] In order to avoid confusions with the Lie brackets "[" and "]", and simultaneously attempting to use a notation frequently employed in linear algebra, we will