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Constrained multi-objective trajectory planning of parallel kinematic machines

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ABSTRACT

This paper presents a new approach to multi-objective dynamic trajectory planning of parallel kinematic machines (PKM) under task, workspace and manipulator constraints. The robot kinematic and dynamic model, (including actuators) is first developed. Then the proposed trajectory planning system is introduced. It minimizes electrical and kinetic energy, robot traveling time separating two sampling periods, and maximizes a measure of manipulability allowing singularity avoidance. Several technological constraints such as actuator, link length and workspace limitations, and some task requirements, such as passing through imposed poses are simultaneously satisfied. The discrete augmented Lagrangean technique is used to solve the resulting strong nonlinear constrained optimal control problem. A decoupled formulation is proposed in order to cope with some difficulties arising from dynamic parameters computation. A systematic implementation procedure is provided along with some numerical issues. Simulation results proving the effectiveness of the proposed approach are given and discussed.

1. Introduction

The design of parallel kinematic machines (PKMs) dates back to the pioneer work by Gough [1], who established the basic principles of a manipulator in a closed loop structure. The machine was able to position and orientate an end-effector (EE), such that to test tire wear and tear. A decade later, Stewart [2] proposed a platform manipulator for the use as an aircraft simulator. Since then, extensive research efforts lead to the realization of several robots and machine tools with parallel kinematic structures [3]. PKMs have two basic advantages over conventional machines of serial kinematic structures. First, the connection between the base and the EE is made with several kinematic chains. This results in high structural stiffness and rigidity. Second, with such structure, it is possible to mount all drives on or near the base. This results in large payloads capability and low inertia. Indeed, the ratio of payload to the robot load is usually about 1/10 for serial robots, while only 1/2 for parallel ones. Despite these advantages, PKMs are still rare in the industry. Among the major reasons for this gap are the small workspace, complex transformations between joint and Cartesian space and singularities as compared to their serial counterparts. These facts lead to a tremendous amount of research in PKMs design and customization [3]. Another reason recently identified is the under consideration of the dynamics of these machines [4]. The mentioned architecture-dependent performance associated with the strong coupled nonlinear dynamics makes the trajectory planning and control system design for PKMs more difficult, as compared to serial machines. In fact, for serial robots, there is a plentiful literature published on the topics of off-line and online programming, from both stand points: computational geometry and kinematics, and optimal control including robots dynamics [5–8]. For PKMs, a relatively large amount of literature is devoted to the computational kinematics and workspace optimization issues. The overwhelm criteria considered for PKMs trajectory planning are essentially design-oriented. These include singularity avoidance and dexterity optimization [9–13]. In Ref. [14], the authors had developed a clustering scheme to isolate and avoid singularities and obstacles for a PKM path planning. A kinematic design and planning method had been described in Ref. [15] for a four-bar planar manipulator mechanism. Another related work was considered in Ref. [16], where it had been shown that a motion planning with singularity-free pose change is possible for PKMs. A variational approach is reported in Ref. [17] for planning a singularity-free minimum-energy path between two end-points for Gough–Stewart platforms. This method is based on a penalty optimization method. Penalty methods, however, have several

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Nomenclature

B	reference frame attached to the center of mass of the base
A	reference frame attached to the center of mass of the end-effector (EE)
A_i, B_i	ith attachment point of leg i on body A and B
$\mathbf{p} = [x\ y\ z]^T$	position vector of the origin of A relative to B in B
$\dot{\mathbf{p}} = [\dot{x}\ \dot{y}\ \dot{z}]^T$	velocity vector of the origin of A relative to B
$\boldsymbol{x}_1 = \mathbf{q} = [\mathbf{p}^T\ \varphi\ \theta\ \psi]^T$	position and orientation of A in B
$\dot{\mathbf{q}}_E = [\dot{\mathbf{p}}^T\ \ \dot{\varphi}\ \ \dot{\theta}\ \ \dot{\psi}]^T$	time derivatives of $\boldsymbol{x}_1(t)$
$\boldsymbol{x}_2 = \dot{q} = [\mathbf{p}^T\ \ \boldsymbol{\omega}^T]^T$	Cartesian and angular velocity of the EE
$\boldsymbol{x} = [\boldsymbol{x}_1\ \boldsymbol{x}_2]^T$	continuous-time state of the PKM
$\boldsymbol{x}_k = [\boldsymbol{x}_{1k}\ \boldsymbol{x}_{2k}]^T$	discrete-time state of the PKM
$\tau(t)$	Cartesian force/torques wrench
$\boldsymbol{i} = [i_1\ i_2 \ldots i_6]^T$	vector of electric currents
$\boldsymbol{l} = [l_1\ l_2 \ldots l_6]^T$	vector of the link lengths
$\boldsymbol{J}$	Jacobian matrix of the PKM
$\boldsymbol{M}_j(\boldsymbol{q})$, $\boldsymbol{M}_c(\boldsymbol{q})$	inertia matrix expressed in joint and Cartesian space
$\boldsymbol{N}_j(\boldsymbol{q}, \dot{\boldsymbol{q}}), \boldsymbol{N}_c(\boldsymbol{q}, \dot{\boldsymbol{q}})$	coriolis and centrifugal force/torque in joint and Cartesian space
$\boldsymbol{G}_j(\boldsymbol{q})$, $\boldsymbol{G}_c(\boldsymbol{q})$	gravity force in joint and Cartesian space
$\boldsymbol{M}_a$, M_a	actuator inertia matrix and its component
$\boldsymbol{V}_a$, V_a	actuator viscous damping coefficient matrix and its component
$\boldsymbol{K}_a$, K_a	actuator gain matrix and its component
$\boldsymbol{K}$	control law gain matrix
τ_m	joint torque vector produced by the DC motors
p	ballscrew pitch
n	gear ratio
J_s, J_m	ballscrew and motor mass moments of inertia
b_s, b_m	ballscrew and motor viscous damping coefficients
$\boldsymbol{\lambda}$	Lagrangian multipliers (or co-states) associated to state variables
$(\boldsymbol{\rho}, \boldsymbol{\sigma})$	Lagrangian multipliers associated to inequality and equality constraints
$(\boldsymbol{\mu}_g, \boldsymbol{\mu}_S)$	penalty coefficients associated to inequality and equality constraints
N	total number of discretisations of the trajectory
w^*, η^*, η^*_1	cost minimization, equality and inequality constraints optimal tolerances
w_t, η_t, η_{t1}	cost minimization, equality and inequality constraints feasible tolerances

drawbacks [18]. Another major issue for off-line programming and practical use of PKMs in industry (in a machining process, for example) is that for a prescribed tool path in the workspace, the control system should guarantee the prescribed task completion within the workspace, for a given set up of the EE (i.e., for which limitations on actuator lengths and physical dimensions are not violated). This problem has been recently considered in Refs. [19,20], with design methodologies involving workspace limitations and actuator forces optimization using optimization techniques.

In this context, we consider a new integrated multi-objective dynamic trajectory planning system for PKMs. Part of this work has been presented in Refs. [21–23]. The proposed approach considers PKM's dynamics, including actuators models as well as task and workspace requirements, as a unique entity. It can be encapsulated into two levels (see Fig. 1): the modeling and approach level and the simulation and testing level [7,23]. The former consists to select according to performance targets related to the robot, task and workspace interactions, the appropriate models and control approaches in order to optimize an overall performance of the robot–task–workspace system. The second level is devoted to coding, testing and validation. Criteria to be optimized in this study are time, energy and a measure of manipulability necessary for a task execution. The optimization procedure is performed within a proper balance between time and energy minimization, singularity avoidance, actuators, sampling periods, link lengths and workspace limitations, and task constraints satisfaction. From a state-space representation by a system of differential equations in the phase plane, the trajectory planning is formulated within a variational calculus framework. The resulting constrained nonlinear programming problem is solved using an augmented Lagrangian (AL) with decoupling technique. AL algorithms have proven to be robust and powerful to cope with difficulties related to none strictly convex constraints [24–27] as compared to optimization methods employing only penalty. The decoupling technique is introduced in order to solve some difficult computations in the original nonlinear and coupled formulation. Another advantage of the proposed method is that one might introduce several criteria and constraints to satisfy in the trajectory planning process.

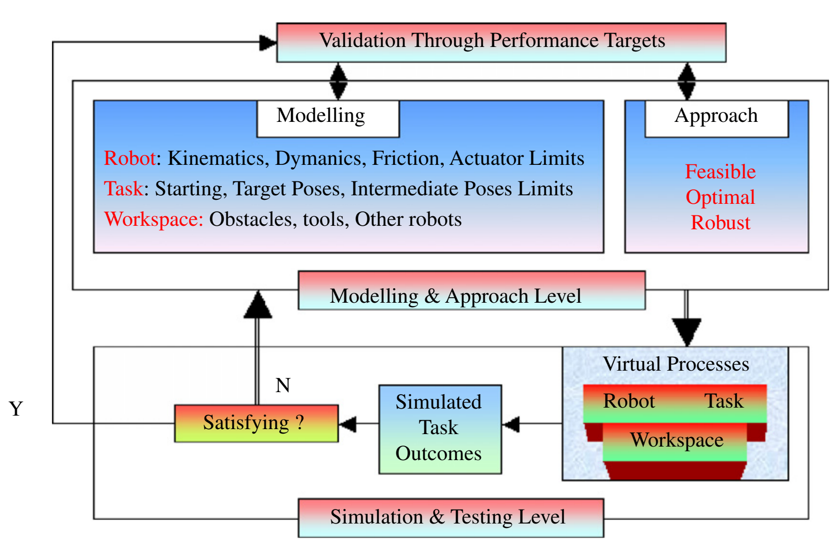

Fig. 1. Overall off-line programming framework of PKMs.