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Experimental validation and comparative analysis of optimal time-jerk algorithms for trajectory planning

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ABSTRACT

In this paper, two minimum time-jerk trajectory planning algorithms for robotic manipulators have been considered, evaluated and experimentally validated. These algorithms consider both the execution time and the integral of the squared jerk along the whole trajectory, so as to take into account the need for fast execution and the need for a smooth trajectory, by adjusting the values of two weights. A comparative analysis of these algorithms with two different trajectory planning techniques taken from the literature has been carried out, by means of experimental tests performed on a real robotic manipulator. The results prove the experimental effectiveness of the proposed techniques.

1. Introduction

In CNC machines and industrial robotic applications there is a growing demand for improving the manufacturing processes and increasing productivity through fast, high precision motion.

This pushes designers and manufacturers towards a reduction of inertia and mass of the whole robotic system, thus resulting in decreased structural stiffness, which in turn causes a considerable increase of vibrational phenomena.

Generation of trajectories with a bounded value of jerk (defined as the derivative of the acceleration) becomes an important issue because this will improve the tracking accuracy and will allow to reach a higher speed of task execution, with eventually a reduction in the excitation of the resonant frequencies of the robot and, consequently, of the mechanical wear of the system (see papers [1–7] for a more detailed description).

Planning a robot trajectory that is optimal with respect to parameters such as jerk and acceleration is thus crucial in order to achieve high-level dynamic performances. To this end, it is desirable to obtain trajectories with continuous accelerations of the joints, so that the absolute value of the jerk keeps bounded. The resulting trajectory will then have the feature of "smoothness" that is required to avoid excitation of the mechanical resonances of the robotic manipulator.

Several optimal off-line trajectory planning techniques can be found in the scientific literature. They may be categorized according to the objective function to be minimized, which is mostly related to execution time, energy or jerk.

Prior work in robot trajectory planning focused mainly on time optimality (unconstrained minimum-time algorithms such as in Refs. [8,9]), due to the need of increasing the productivity in the industrial sector. More recently, minimum time algorithms under kinematic constraints (such as the maximum allowed values of velocity, acceleration and jerk) have been considered [10–15].

For instance, in Ref. [11] Dong et al. present a method for determining time optimal path-constrained trajectories subject to velocity, acceleration and jerk constraints acting on both the manipulator actuators and on the task to be executed. The optimization problem is solved using a hybrid optimization strategy, starting from the path description, the kinematic relations of the manipulator and the defined constraints. However, the resulting trajectories are optimal with respect to time, not with respect to smoothness.

Another optimality criterion for trajectory generation is the minimization of the energy required to the robot actuators. Several energy-optimal algorithms were proposed in the scientific literature [16–19]. Minimizing the energy consumption instead of the execution time provides several advantages: the resulting trajectories are easier to track and allow to reduce the stresses to the actuators and to the robot structure. Moreover, minimum-energy trajectories are strongly recommended in some applications, for instance when the capacity of the energy source is limited (e.g. robots for spatial or submarine exploration).

Hybrid time-energy-optimal trajectory planning techniques were also proposed in the scientific literature (see for instance Refs. [20–22]).

Jerk-optimal algorithms constitute another category of trajectory planning techniques [3–5]. As mentioned in the foregoing,

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optimality with respect to jerk enables one to educe the excitation of the resonant frequencies of the mechanical system, thus reducing the tracking errors, the stresses to the actuators and to the manipulator structure. Recently, hybrid time-jerk optimal trajectories were proposed [23–28], so as to reach the advantages of the jerk reduction while planning fast trajectories. In Ref. [23] trigonometric splines are used to interpolate the trajectory ensuring the jerk continuity, while in Ref. [24] a method based on genetic algorithms is developed for planning a global minimum-jerk trajectory. An important result in jerk-optimal trajectory planning is represented by the work by Piazzi and Visioli [4,5] who develop an algorithm, based on interval analysis, that globally minimizes the maximum absolute value of the jerk along a trajectory whose execution time is set *a priori*. The trajectories are expressed by means of cubic splines and the intervals between them via points are computed, so that the lowest jerk peak is produced. In Refs. [27,28] two algorithms for time-jerk optimal trajectory planning for robot manipulators are proposed, based upon a minimization of an objective function composed of two terms, one proportional to the total execution time and one proportional to the integral of the squared jerk along the trajectory. These terms are weighted by two different parameters that in Refs. [27,28] are empirically chosen.

Another hybrid technique is that in Refs. [29,30], where a time-jerk-energy optimal trajectory planning algorithm is presented. This method is based on the objective function defined in Refs. [27,28], which was extended by considering also the joint physical limits and the instantaneous power consumption of the actuating motors.

Most of the trajectory planning techniques found in the scientific literature were not experimentally tested, although the experimental validation of a trajectory planning algorithm would be of great interest. To the authors' knowledge, only in Refs. [6,7,10,29–32] experimental tests were proposed, either on manufacturing and on robotic systems, for instance Refs. [7,31,32] by measuring the acceleration of a specific part of the system.

In this paper, the trajectory planning algorithms proposed in Refs. [27,28] is considered. First, a suitable criterion for choosing the weights for the time and the jerk terms is defined; then, the algorithms are executed and the resulting trajectories are input to a 3 degrees of freedom Cartesian manipulator in order to have an experimental validation. These algorithms enable one to define constraints on the robot motion, which are expressed as upper bounds on the absolute values of velocity, acceleration and jerk for all robot joints, so that any physical limitation of the real manipulator can be taken into account when planning the optimal trajectory. Moreover, unlike most jerk-minimization methods, the considered techniques do not require an *a priori* setting of the total execution time.

In order to perform a comparative analysis of the experimental results, both a classical spline based planning algorithm and Piazzi and Visioli's technique [4,5] have also been implemented, input to the Cartesian robot and experimentally tested (to the authors' knowledge, this is the first time Piazzi and Visioli's algorithm has been experimentally evaluated and tested).

The paper is organized as follows: the two minimum time-jerk trajectory planning algorithms and a suitable criterion for choosing the weights for the time and jerk terms of the objective function are described in Section 2. In Section 3 Piazzi and Visioli's global minimum jerk algorithm is synthetically recalled; in Section 4 the main characteristics of all the trajectory planning techniques under test are analyzed and critically compared. Section 5 describes the experimental set up, Section 6 deals with the simulation, the experimental tests and the comparative analysis of the results.

2. Minimum time-jerk trajectory planning algorithms

The minimum time-jerk trajectory planning techniques, presented in Refs. [27,28] provide *off-line* trajectories, starting from a geometric path generated by a top level planner. The path is given as a sequence of via-points in the operative space representing the position and the orientation of the robot end effector, and it is then transformed into a sequence of via-points in the joint space, by means of a kinematic inversion. The considered algorithms find an optimal trade-off between jerk and execution time and, unlike most minimum jerk trajectory planning techniques found in the scientific literature, do not require to impose the execution time *a priori*. Constraints on the robot joints, such as upper bounds on velocity, acceleration and jerk, are taken into account while executing the algorithms.

Both minimum time-jerk trajectory planning techniques are based on the same optimization problem, which is formulated as:

$$\begin{cases} find: \\ min k_T \cdot N \sum_{i=1}^{vp-1} h_i + k_J \sum_{0}^{N} \int_{j=0}^{tf} q_j(t)^2 dt \\ subject\ \ to: \\ \left|\dot{q}_j(t)\right| \le VC_j & j = 1\ldots N \\ \left|\ddot{q}_j(t)\right| \le WC_j & j = 1\ldots N \\ \left|q_j(t)\right| \le JC_j & j = 1\ldots N \end{cases} \quad (1)$$

The first term of the objective function that appears in (1) is proportional to the trajectory execution time, while the second one is proportional to the integral of the squared jerk. The two effects are weighted by the coefficients k_T and k_J, respectively, and a balance between speed and smoothness can be reached by suitably choosing the two weights between the two limit conditions, namely the minimum jerk trajectory ($k_T = 0$) and the minimum time trajectory ($k_J = 0$).

In Table 1 the meaning of the symbols of Eq. (1) are explained.

The solution of the system (1) is a vector containing the values of the time intervals h_i, between two consecutive via-points, which minimize the objective function.

Specific primitives have to be chosen, in order to represent the trajectories. Two algorithms, both based upon the objective function in (1), were designed, namely a cubic splines based technique (named SPL3J) and a fifth-order B-splines based technique (named BSPL5J).

2.1. SPL3J algorithm

For a trajectory made of cubic splines, let $Q_{j,i}(t)$ be the cubic polynomial for the jth joint defined on the interval $[t_i, t_{i+1}]$.

Table 1
Meaning of symbols used in Eq. (1).

N	Number of robot joints
k_T	Weight of the term proportional to the execution time
k_J	Weight of the term proportional to the jerk
vp	Number of via-points
h_i	Time interval between two via-points
tf	Total execution time of the trajectory
$\dot{q}_j(t)$	Velocity of the jth joint
$\ddot{q}_j(t)$	Acceleration of the jth joint
$q_j(t)$	Jerk of the jth joint
VC_j	Velocity bound for the jth joint (symmetrical)
WC_j	Acceleration bound for the jth joint (symmetrical)
JC_j	Jerk bound for the jth joint (symmetrical)