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Recursive modelling in dynamics of Agile Wrist spherical parallel robot

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Abstract

Recursive matrix relations for kinematics and dynamics of the 3-*RRR* Agile Wrist spherical parallel robot are established in this paper. The prototype of the robot is a three-degrees-of-freedom mechanism with three identical legs. Controlled by concurrent torques, which are generated by some electric motors, the active elements of the robot have three independent rotations. Knowing the rotation motion of the moving platform, we develop first the inverse kinematical problem and determine the velocities and accelerations. Further, the principle of virtual work is used in the inverse dynamic problem. Matrix equations offer iterative expressions and graphs for the power requirement comparison of each of three actuators in two computational complexities: complete dynamic model and simplified dynamic model.

Keywords: Dynamics; Kinematics; Parallel robot; Virtual work

1. Introduction

Parallel manipulators are closed-loop mechanisms that consist of separate serial chains connecting the fixed base to the moving platform.

Typically, a parallel mechanism is said to be *symmetrical* if it satisfies the following conditions: the number of legs is equal to the number of degrees of freedom (DOFs) of the moving platform, one actuator controls every limb and the location and the number of actuated joints in all the limbs are the same [1].

The parallel robots can be equipped with hydraulic or pneumatic actuators. They have a robust construction and can move bodies of considerable masses and dimensions with high speeds. This is why the mechanisms, which produce a translation or spherical motion to a platform, are based on the concept of parallel manipulator.

Parallel manipulators attracted to the attention of more and more researches that consider them as valuable alternative design for robotic mechanisms [2,3]. Compared with the serial robots, parallel mechanisms have some special characteristics: greater structural rigidity, better orientation accuracy, functional stability, larger dynamic charge capacity and suitable position of the actuating systems. On the other hand, parallel kinematics machines offer essential advantages over serial robots: lower moving masses, higher natural frequencies, simpler modular mechanical construction and possibility to mount all actuators at or near the fixed base. However, most existing parallel robots have limited and complicated workspace volume with singularities and highly non-isotropic input–output relations [4].

Recently, many efforts have been assigned to kinematics and dynamic analysis of fully parallel manipulators. Many companies have developed them as high precision machine tools. Among these, the class of manipulators known as Stewart–Gough platform focused great attention [5–7]. They are used in flight simulators, pointing devices, and more recently, for parallel kinematic machines. The Star parallel manipulator [8] and the Delta parallel robot [9–11] are equipped with three motors and train on the mobile platform in a three DOF general translation motion. Angeles [4] and Gosselin and Gagné [12] analysed the direct kinematics, dynamics and singularity loci of the Agile Wrist prototype that presents three concurrent rotations.

The kinematics and dynamics of parallel structures have been studied extensively during the last two decades.

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Nomenclature

$a_{k,k-1}$	orthogonal transformation matrix
R	rotation matrix of the moving platform
$\alpha_1,\alpha_2,\alpha_3$	Euler angles giving the orientation of the platform
$\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$	three orthogonal unit vectors
$\alpha_A,\alpha_B,\alpha_C$	angles giving the position of three actuators
r,β,γ	radius and angles of proximal and distal links
$\varphi_{k,k-1}$	relative rotation angle of T_k rigid body
$\vec{\omega}_{k,k-1}$	relative angular velocity of T_k
$\vec{\omega}_{k0}$	absolute angular velocity of T_k
$\tilde{\omega}_{k,k-1}$	skew-symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$
$\vec{\varepsilon}_{k,k-1}$	relative angular acceleration of T_k
$\vec{\varepsilon}_{k0}$	absolute angular acceleration of T_k
$\tilde{\varepsilon}_{k,k-1}$	skew-symmetric matrix associated to the angular acceleration $\vec{\varepsilon}_{k,k-1}$
$\vec{r}^{A}_{k,k-1}$	relative position vector of the centre A_k of joint
m_k	mass of T_k rigid body
$\hat{J}_k$	symmetric matrix of tensor of inertia of T_k about the link-frame $x_k y_k z_k$
m^A_{10}, m^B_{10}, m^C_{10}	torques of three actuators pointing about z^A_1, z^B_1, z^C_1 directions
p^A_{10}, p^B_{10}, p^C_{10}	powers developed by the actuators

When a good dynamic performance and a precise positioning of the moving platform under high load are required, the dynamic model of the robot is important for its automatic control [13].

Meanwhile, quite few of special approaches have been conducted for dynamic modelling of specific parallel mechanism configurations. Sorli et al. [14] conducted the dynamics modelling for Turin parallel manipulator, though the mechanism has three identical legs, it has 6-DOFs. Geng et al. [15] developed Lagrange's equations of motion under some simplifying assumptions regarding the geometry and inertia distribution of the manipulator. Dasgupta and Mruthyunjaya [16] used the Newton–Euler approach to develop closed-form dynamic equations of Stewart platform, considering all dynamic and gravity effects as well as viscous friction at joints.

A space parallel mechanism, which can be used in several applications including machining tools, is proposed in the paper. We focus our attention on an efficient and fast recursive matrix method, which is adopted to derive the kinematical model and the inverse dynamics equations of the 3-DOF symmetric spherical parallel robot with revolute actuators (Fig. 1).

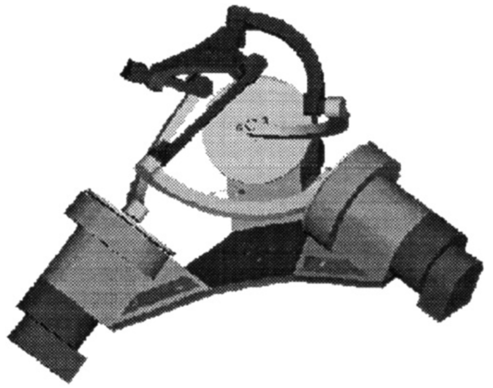

Fig. 1. Spherical parallel robot.

2. Inverse kinematics

Spherical parallel manipulators provide high stiffness and accuracy due to their mechanical arrangement in which all the actuators are fixed to the base and the end-effector is supported by three kinematical chains.

Parallel spherical wrists are used in technical applications where it is desired to orient a rigid body at high speed. Accuracy and precision in the execution of the task of the wrist are essential since the robot is intended to operate on various objects; where positioning errors of the tool could end in costly damage. Example of such applications includes the orientation of a camera at high velocity.

This robot has a structure with the axes of all nine revolute joints concurring in a common *centre O of rotation* of the device. The mechanism input of the robot is structured into three active revolute joints while the output body is connected through a set of legs with identical topology.

Let $Ox_0y_0z_0(T_0)$ be a fixed Cartesian frame, about which the manipulator moves. It has three legs of known size and mass (Fig. 2). The wrist architecture consists of two main elements, the base and the moving platform, which is free to undergo arbitrary rotations with respect to the centre of three identical serial legs, each of these composed of two links coupled each other by means of a revolute joint.

To simplify the graphical image of the kinematical scheme of the mechanism, in the follows we will represent the intermediate reference systems by only two axes, so as one precedes in most of robotics papers [1,3,4,7]. The z_k axis is represented, of course, for each component element T_k. It is noted that the relative rotation with $\varphi_{k,k-1}$ angle of T_k body most be always pointing about the direction of the z_k axis. The first element T_1 of leg A, one of the three driving parts of the robot, is called *proximal link*. It is a homogenous rod, rotating about the axis Oz^A_1 with the angular velocity $\omega^A_{10} = \dot{\varphi}^A_{10}$ and the angular acceleration $\varepsilon^A_{10} = \ddot{\varphi}^A_{10}$. It has the radius r, masse m_1 and tensor of inertia $\hat{J}_1$. The *distal link* T_2 of radius r is connected to the $A_2x^A_2y^A_2z^A_2$ frame and has a relative rotation with the angle