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Tool-path generation of planar NURBS curves

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ABSTRACT

It has been widely used in CAD field for many years and gradually applied in CAM area with the prevalence of NURBS interpolator equipped in CNC controllers. But few of them provide the tool radius compensation function. In order to achieve the goal of generating tool-path, an algorithm was presented to offset NURBS curves by an optimum process for CAD/CAM systems in this paper. NURBS format is ideal for HSM applications, but not all NURBS outputs are equal and standard. Basically, there are two different ways to generate NURBS tool-paths; one is to fit a NURBS curve to the conventional tool-path output, the other one is to generate a NURBS tool-path from the start. The main targets for the tool-path of this paper are: (1) To keep a constant distance d between progenitor curve $C(t)$ and offset curve $C_d(t)$ on the normal direction of $C(t)$; (2) to alternate the order k of the basis function in offset curve $C_d(t)$; (3) to oscillate the number of control points of offset curve $C_d(t)$ and compare it with progenitor curve $C(t)$. In order to meet the tolerance requirements as specified by the design, this study offsets the NURBS curves by a pre-described distance d. The principle procedure consists of the following steps: (1) construct an evaluating bound error function; (2) sample offset point-sequenced curves based on first derivatives; (3) give the order of NURBS curve and number of control points to compute all initial conditions and (4) optimize the control points by a path searching algorithm.

1. Introduction

To meet the recent demands for low cost and high speed machining (HSM), the processing time has to be shortened as much as possible. Conventionally, the typical way to perform the cutter location (CL) paths is approximated with piecewise line or circular segments by CAM systems. The use of this approximation approach has some inherent disadvantages: (1) to satisfy the machining accuracy requirement, the data size of the NC program is often relatively large and therefore results in long transmission loading; and (2) the cutter needs to accelerate and decelerate at each line segment, which leads to a velocity discontinuity at the junction of two connected lines segments [1]. NURBS-based tool-paths expand the traditional linear and circular codes with a spline represented by control points and knot vectors. B-spline machining has several advantages; the tolerance can be minimized when B-splines are fitted to straight line cutter paths. Tool movements are more consistent, which reduces dwell marks. The file size shall be reduced and fewer points are required to define a curve. In essence, NURBS interpolation typically results in shorter cycle times, smaller programs, more accurate parts and better profile finishes. Besides look-ahead features in the CNC controller dynamically change the feed rate, slowing the spindle to allow rapid changes in direction. A remarkable amount of NC data is saved by this function, which can result in fluent HSM.

It has been widely used in CAD field for many years and is gradually applied in CAM area with the prevalence of NURBS interpolator equipped in CNC controllers. But few of them provide tool radius compensation function. In order to achieve the goal of generating tool-path, an algorithm is presented to offset NURBS curves by an optimum procedure. Recently, Lai et al. [2] proposed an optimum process to reduce the degree of NURBS curves. Following the optimum process, this paper describes a radiating-web like searching path and a normalizing process to find the offsets of planar NURBS curves. The main targets of this paper are: (1) To keep a constant distance d between offset curve $C_d(t)$ and progenitor curve $C(t)$ on the normal direction of $C(t)$. (2) To alternate the order k of the basis function in offset curve $C_d(t)$. Generally, reducing of the order is better for calculating the initial conditions. (3) To inherit the same number of control points from progenitor curve $C(t)$.

Instead of 3D curves, this paper develops planar NURBS paths for speeding up the feed-rate to 100 M/min in practical application. Such a high speed machining will be applied on an appropriate working table. Usually, Z-axis will restrict this operation for the short stroke. Today ball-screw systems are available with specifications from 0.1 to 1.5 g, whereas linear motors typically accelerate at rate of 2 g, even more. A machine center with FANUC 16 M controller, NURBS-based tool-paths, high

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precision contour control (HPCC) and linear motors [3] can cut a large shape of working piece ($3000 \times 1500\,\text{mm}^2$) completely in 10 s.

Traditionally, there are three different directions to develop offset problem on free-form curves [4–10]. (1) Approximate the progenitor curve by piecewise straight lines and circular arcs first and offset these entities precisely. This method cannot inherit the original free-form curve's properties. (2) Offset the control polygon that consists of control points with the same degree and weights as the original one. This method is fast but the accuracy problem exists in most of free-form cases, and it works only on convex patterns. Occasionally, this method may perform well while the control points are spread regularly and curvature deviation is small. (3) Most of the approximate schemes use a subdivision technique to offset a free-form curve. The main flaw in this method is that it may increase too many control points to meet a prescribed tolerance. This method described in this paper will improve the above statements.

An n side open shape consist of (n+1) control points, thus (n+1) control points can form a B-spline curve, which is defined by the following equation:

$$C(t) = \sum_{i=0}^{n} N_{i,k}(t)P_i$$

A NURBS curve is defined by the following equation:

$$C(t) = \frac{\sum_{i=0}^{n} N_{i,k}(t)w_iP_i}{\sum_{j=0}^{n} N_{j,k}(t)w_j} = \sum_{i=0}^{n} R_{i,k}(t)P_i, \quad R_{i,k}(t) = \frac{N_{i,k}(t)w_i}{\sum_{j=0}^{n} N_{j,k}(t)w_j} \tag{1}$$

where P_i are vectors composed of x and y coordinates of the control points, w_i are weights for each point, $N_{i,k}(t)$ are B-spline basis functions. k (k=p+1, p is the degree of basis function. For example, when order is 3, then rank in the definition of NURBS is 2, i.e. the NURBS expression is expressed by t^2, t and constant) is the order of a B-spline curve. The basis function is expressed by de Boor–Cox as follows:

$$N_{i,1}(t) = \begin{cases} 1 & \text{if}(t_i \le t < t_{i+1}) \\ 0 & \text{(otherwise)} \end{cases}$$

$$N_{i,k}(t) = \frac{t-t_i}{t_{i+k+1}-t_i} N_{i,k-1}(t) + \frac{t_{i+k}-t}{t_{i+k}-t_{i+1}} N_{i+1,k-1}(t) \tag{2}$$

$[t_i]$, i=0~(n+k), $t_i \le {}_{i+1}$, is the knot vector.

The derivative of a B-spline curve is

$$C'(t) = \sum_{i=0}^{n} N'_{i,k}(t)P_i \tag{3}$$

$$N_{i,1}(t) = 0$$

$$N'_{i,k}(t) = \frac{N_{i,k-1}(t) + (t-t_i)N'_{i,k-1}(t)}{t_{i+k+1}-t_i} + \frac{(t_{i+k}-t)N'_{i+1,k-1}(t) - N_{i+1,k-1}(t)}{t_{i+k}-t_{i+1}}$$

The derivative of a NURBS curve is

$$C'(t) = \sum_{i=0}^{n} R'_{i,k}(t)P_i, \quad w(t) = \sum_{j=0}^{n} N_{j,k}(t)w_j, \quad A_i(t) = N_{i,k}(t)w_i \tag{4}$$

$$R_{i,k}(t) = \frac{A_i(t)}{w(t)}, \quad R'_{i,k}(t) = \frac{w(t)A'_i(t) - w'(t)A_i(t)}{w(t)^2}$$

If a series of control points exist, one can refine the shape of such a curve in the following different ways. They offer a wide range of tools to design and analyze shape deformation. They are:

Moving of the control points P_i.
Increasing or decreasing the number of control points.
Multiple locating of the control points.
Changing the order k of the basis functions.
Modifying the weights w_i for each control point.
Replacing the basis function (closed uniform, open uniform and non-uniform).
Rearranging the knot vector $[t_i]$.
Using multiple knot values in knot vector.
Changing the relative spacing of the knots.

Furthermore, one can use dense control points to meet a prescribed arbitrary tolerance and get better performance directly. Unfortunately, one of the main concerns is to retain the same number of control points. There are only a few papers published on offsets of planar NURBS curves. The main reason may be the varying distributions of weight at each control point. It is hard to construct an approximating equation to represent an offset curve directly. Local control is one of the geometrical advantages of NURBS curves. By changing one control point, one can adjust the relative shape portion. Based on this advantage, this study regards location as a parameter to perturb control points and detect optimum positions globally. This study proposes a simpler but more intuitive method to find the optimum solutions for offsets of NURBS curves by using several different examples to verify the precision and evaluate the practicability of our method. From different curvature variations, offset curves may perform different convergence processes and affect the optimum solutions.

This paper is organized as follows: Section 2 reviews several previous works on free-form curve offsets. Section 3 defines a proposed evaluating bound error function, which consists of offset distance deviation and angle variation. Section 4 provides a brief introduction of the regularization parameter and normalization process. In Section 5, a radiating-web like searching path is proposed to detect optimum solutions. And the test results are verified by the proposed algorithm in Section 6. Finally, the conclusion is proposed in Section 7.

2. Previous works

Due to the widespread use of NURBS-based curve representation and motion control, the demand for high-speed machining (HSM) is increasing. A real part can be made by the corresponding cutting paths guided by tool-radius compensation interpolator. In addition, these operations are based on parametric curves offset algorithms.

Fig. 1 illustrates the cutting task of NURBS-based CNC machining [11]. In modern CAD/CAM systems, more and more profiles are represented in parametric forms to meet the requirement of HSM. Many researchers have developed real-time parametric interpolators for curve generation using NURBS basis function [1,12–15]. Some major commercial controller manufacturers have brought NURBS operations into industry, such as Fanuc 15 M/16 M [16] and Siemens 840D [17]. Once the spline format is sent to the controller, the processor directly interpolates the segments at extremely tiny intervals. The architecture of the controller has look-ahead features that will change the feed-rate dynamically to adapt the spindle to rapid changes in direction.

New developments in high-speed machining have been possible because of the improvements in CAM. Having said that, a balanced approach is essential in which all aspects are considered, including cutters, spindles, CAM software, etc. High-speed machining is a process, and software is an integral component of the process. The effective operation requires a machine tool, controller and programming software that are all designed to support the high-speed machining process. The