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Static analysis of spatial parallel manipulators by means of the principle of virtual work

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ABSTRACT

It is presented a comprehensive approach for the static analysis of spatial parallel manipulators using the principle of virtual work, equipped with a recursive and systematic formulation, which is intended for conducting an efficient manipulation of the kinematics associated with the problem. Thus, it is possible omitting all internal forces and nonworking external constraint forces in the problem formulation. As a result, the actuator drive forces and/or torques can be directly related with the external loads supported by the manipulator, including the weight of the mobile platform and also the weight of the links of the connecting legs. A thorough understanding of these forces and/or torques is important for proper sizing of actuators at the design stage. In order to prove the feasibility and the validity of the proposed method, two fully detailed examples are presented.

1. Introduction

One of the most important subjects of statics is directed towards the quantitative description of loads[1] that act on mechanical systems in equilibrium. For a mechanical system, *static equilibrium* is described as the state where all components of the system are at rest, with zero velocity and zero acceleration [1,2].

Static force analysis is of practical importance in determining the quality of force transmission through the various joints of a mechanism. It serves as a basis for sizing the links and bearings of a robot manipulator and for selecting appropriate actuators. Thus, a static analysis could not only reduce in a significative manner the computational effort usually required by the dynamic analysis of a given manipulator, but also it could be used to obtain a lower bound of the forces demanded at the actuators. Moreover, under the assumption that the mobile platform performs slow motions, the dynamics of the parallel manipulator is dominated by the gravity forces. Furthermore, transient dynamic analysis of complex mechanical systems is often initiated from a configuration of static equilibrium [1].

It is well known that the statics of parallel manipulators is more complicated than statics of serial manipulators. This is mainly due to the existence of several closed loops. Moreover, in general, it is necessary to derive the force and moment balance equations for each link and solve the equations simultaneously. However, if only the actuator drive forces and/or torques are of interest, the principle of virtual work can be applied. The principle of virtual work is a very powerful tool since the reactions and internal forces are not involved in the formulation, and the number of equations is greatly reduced. As an important result, the actuator drive forces and/or torques can be directly related with the external loads supported by the manipulator. A thorough understanding of these forces and/or torques is important for proper sizing of actuators at the early design stages of the manipulator.

On the other hand, it should be noted that D'Alembert's principle [2] can be used in order to extend the principle of virtual work from the static to the dynamic case. There, the negative of the rate of change of linear and angular momentum are treated as force and moment, respectively, referred as *inertia force* and *inertia moment* that provide the equilibrium of the mechanical system under study. Therefore, it is important to define what kind of analysis is going to be performed. Thus, regarding the principle of virtual work, research works concerning with the static case have been reported in [3,4], whereas Refs. [5–9] consider the dynamic case. More specifically, Refs. [3,4], just show the static analysis for a particular parallel manipulator, the 3-UPU spatial manipulator. Nothing is said about a systematic procedure to perform the static analysis of arbitrary spatial parallel manipulators.

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[1] For brevity, through this paper we will use the term *load* to imply both, *force* and *moment*.

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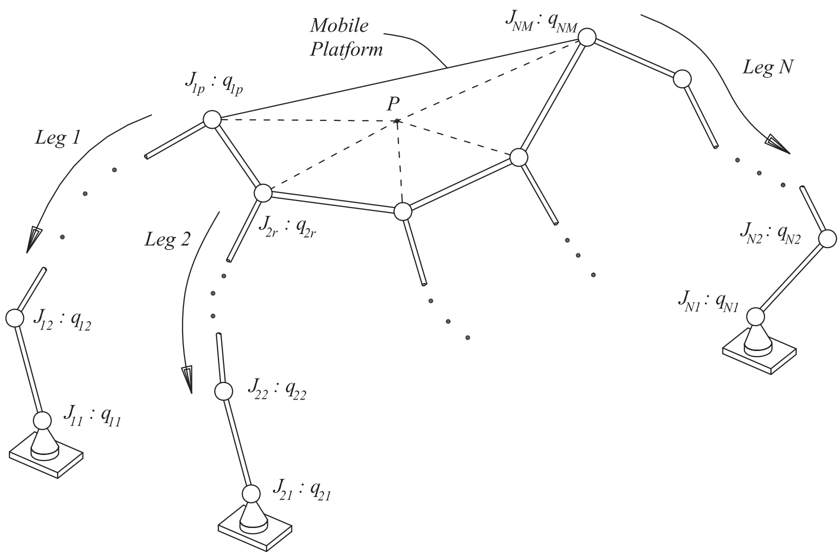

Fig. 1. Kinematic notation for an arbitrary parallel manipulator.

Furthermore, it should be also noted that there exists a very simple equation [10] which involves the end-effector external forces and moments, **F**, the actuated joint forces and/or torques, **f**, and the Jacobian matrix of the manipulator, **J**, namely

$$\mathbf{F} = \mathbf{J}^T \mathbf{f} \tag{1}$$

which was derived by resorting to the principle of virtual work applied on the static analysis of an arbitrary parallel manipulator. However, it is important to realize that Eq. (1) emphatically assumes that the gravitational effects, namely, the weights[2] of the links, are negligible [10]. Such assumption is not a desirable feature for a spatial parallel manipulator composed of links with large masses and equipped with a heavy mobile platform.

This paper presents a novel methodology especially developed for obtaining the actuated joint forces and/or torques, which are required to maintain the static equilibrium of a spatial parallel manipulator. The manipulator will be subjected to gravitational forces acting on the links, and also to external forces and moments applied on the mobile platform. The proposed approach uses the principle of virtual work, equipped with a recursive and systematic formulation, which is intended for conducting a systematic manipulation of the kinematics associated with the problem. As a result, the proposed approach can be used for an arbitrary parallel manipulator. It is expected that this paper may contribute to a deeper insight into the significative effects of an adequate kinematic formulation in the static analysis of spatial manipulators.

2. Kinematics of parallel manipulators

A spatial parallel manipulator involves one or more closed loops. This fact increases the complexity of its static analysis. To this end, we found that an efficient solution of the static problem needs to be carried out in two major steps, the first being the kinematic formulation, and the second the application of the principle of virtual work. Since the second step can not be taken fruitfully until the first has been completed, it is imperative a careful and systematic formulation of the kinematic aspects of the problem at hand. Such a formulation will be very significant for a successful completion of the static analysis presented later in the following sections. Moreover, being highly systematic, this methodology is easy to apply to arbitrary parallel manipulators.

2.1. Description of an arbitrary parallel manipulator

In order to characterize the kinematic structure and the joint variables associated with an arbitrary parallel manipulator, we will refer to Fig. 1.

Thus, according with the graphic information shown in Fig. 1, an arbitrary parallel manipulator will be composed of N legs, which provide the relative motion that exists between the mobile platform and the fixed platform. Each leg—a serial chain—may contain a different number of 1-DOF joints. For example, leg 1 involves p1-DOF joints, $J_{11}, J_{12}, \ldots J_{1p}$, whose joint motions—angular or linear—are represented by joint variables q_{11}, q_{12}, $\ldots, q_{1p}$. In this way, the whole set of joint variables is given by

$$\mathbf{q} \equiv (q_{11},\ q_{12}, \ldots, q_{1p},\ q_{21},\ q_{22}, \ldots,\ q_{2r}, \ldots,\ q_{N1},\ q_{N2}, \ldots, q_{NM})^T \tag{2}$$

whose total number will depend on the number of legs and the number of 1-DOF joints composing each leg.

2.2. Velocity state of the mobile platform

The velocity state of a rigid body contains all the first-order (velocity) properties [11] associated with the motion of the body. Thus, the velocity state of the mobile platform with respect to the fixed platform is given by

$$\mathbf{V} = \begin{bmatrix} \boldsymbol{\omega}_{M/F} \\ \mathbf{v}_{P/O} \end{bmatrix} \tag{3}$$

where $\boldsymbol{\omega}_{M/F}$ is the angular velocity vector of the mobile platform with respect to the fixed platform and $\mathbf{v}_{P/O}$ is the velocity vector of an arbitrary point P—fixed to the mobile platform—with respect to an arbitrary point O, which is attached to the fixed platform.

It is important to realize that the velocity state of the mobile platform can be reached from any leg composing the parallel manipulator. Thus, for leg j, it can be obtained that [12]

$$\mathbf{V}_j = \mathbf{S}_{j1}\dot{q}_{j1} + \mathbf{S}_{j2}\dot{q}_{j2} + \cdots + \mathbf{S}_{jp}\dot{q}_{jp} \tag{4}$$

[2] It should be noted that only the weight of the mobile platform could be included into the vector associated with the end-effector external loads, which are grouped into vector **F**.