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Computing the L_1 geodesic diameter and center of a simple polygon in linear time ☆

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ABSTRACT

In this paper, we show that the L_1 geodesic diameter and center of a simple polygon can be computed in linear time. For the purpose, we focus on revealing basic geometric properties of the L_1 geodesic balls, that is, the metric balls with respect to the L_1 geodesic distance. More specifically, in this paper we show that any family of L_1 geodesic balls in any simple polygon has Helly number two, and the L_1 geodesic center consists of midpoints of shortest paths between diametral pairs. These properties are crucial for our linear-time algorithms, and do not hold for the Euclidean case.

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1. Introduction

Let P be a simple polygon with n vertices in the plane. The *diameter* and *radius* of P with respect to a certain metric d are very natural and important measures of P. The diameter with respect to d is defined to be the maximum distance over all pairs of points in P, that is, $\max_{p,q \in P} d(p,q)$, while the radius is defined to be the min–max value $\min_{p \in P} \max_{q \in P} d(p,q)$. Here, the polygon P is considered as a closed and bounded space and thus the diameter and radius of P with respect to d are well defined. A pair of points in P realizing the diameter is called a *diametral pair*. Similarly, any point c such that $\max_{q \in P} d(c,q)$ is equal to the radius is called a *center*. In this paper we study how fast can we compute these measures (and whenever possible, to also obtain the set of points that realize them).

One of the most natural metrics on a simple polygon P is induced by the length of the Euclidean shortest paths that stay within P, namely, the *(Euclidean) geodesic distance*. The problem of computing the diameter and radius of a simple polygon with respect to the geodesic distance has been intensively studied in computational geometry since the early 1980s. The diameter problem was first studied by Chazelle [6], who gave an $O(n^2)$-time algorithm. The running time was afterwards improved to $O(n \log n)$ by Suri [21]. Finally, Hershberger and Suri [11] presented a linear-time algorithm based on a fast matrix search technique. Recently, Bae et al. [3] considered the diameter problem for polygons with holes.

The first algorithm for finding the Euclidean geodesic radius was given by Asano and Toussaint [2]. In their study, they showed that any simple polygon has a unique center, and provided an $O(n^4 \log n)$-time algorithm for computing it. The running time was afterwards reduced to $O(n \log n)$ by Pollack, Sharir, and Rote [17]. Since then, it has been a longstanding open problem whether the center can be computed in linear time (as also mentioned later by Mitchell [14]).

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Table 1
Summary of currently best results on computing the diameter and radius of a simple polygon P with respect to various metrics on P.

	Metric	Restriction on P	Diameter		Radius	
Geodesic	Euclidean	simple	$O(n)$	[11]	$O(n\log n)$	[17]
	L_1	rect. simple	$O(n)$	[18]	$O(n)$	[18]
		simple	$O(n)$	[Theorem 18]	$O(n)$	[Theorem 24]
Link	regular	simple	$O(n\log n)$	[20]	$O(n\log n)$	[8,13]
	rectilinear	rect. simple	$O(n)$	[15]	$O(n)$	[16]

Another popular metric with a different flavor is the *link distance*, which measures the smallest possible number of links (or turns) of piecewise linear paths. The currently best algorithms that compute the link diameter or radius run in $O(n\log n)$ time [8,13,20]. The *rectilinear link distance* measures the minimum number of links when feasible paths in P are constrained to be rectilinear. Nilsson and Schuierer [15,16] showed how to solve the problem under the rectilinear link distance in linear time.

In order to tackle the open problem of computing the Euclidean geodesic center, we investigate another natural metric: the L_1 metric. To the best of our knowledge, only a special case where the input polygon is rectilinear has been considered in the literature. This result is given by Schuierer [18], where he showed how to compute the L_1 geodesic diameter and radius of a simple rectilinear polygon in linear time.

This paper aims to provide a clear and complete exposition on the diameter and radius of general simple polygons with respect to the L_1 geodesic distance. We first focus on revealing basic geometric properties of the geodesic balls (that is, the metric balls with respect to the L_1 geodesic distance). Among other results, we show that any family of L_1 geodesic balls has Helly number two (see Theorem 11). This is a crucial property that does not hold for the Euclidean geodesic distance, and thus we identify that the main difficulty of the open problem lies there.

We then show that the method of Hershberger and Suri [11] for computing the Euclidean diameter extends to L_1 metrics, and that the running time is preserved. However, the algorithms for computing the Euclidean radius do not easily extend to rectilinear metrics. Indeed, even though the approach of Pollack et al. [17] can be adapted for the L_1 metric, the running time is $O(n\log n)$. On the other hand, the algorithm of Schuierer [18] for rectilinear simple polygons heavily exploits properties derived from rectilinearity. Thus, its extension to general simple polygons is not straightforward either.

In this paper we use a different approach: using our Helly-type theorem for L_1 geodesic balls, we show that the set of points realizing L_1 geodesic centers coincides with the intersection of a finite number of geodesic balls. Afterwards we show how to compute their intersection in linear time. Table 1 summarizes the currently best results on computing the diameter and radius of a simple polygon with respect to the most common metrics, including our new results.

2. Preliminaries

For any subset $A \subset \mathbb{R}^2$, we denote by ∂A and $\operatorname{int} A$ the boundary and the interior of A, respectively. For $p, q \in \mathbb{R}^2$, denote by $\overline{pq}$ the line segment with endpoints p and q. For any path π in $\mathbb{R}^2$, let $|\pi|$ be the length of π under the L_1 metric, or simply the L_1 *length*. Note that $|\overline{pq}|$ equals the L_1 distance between p and q.

The following is a basic observation on the L_1 length of paths in $\mathbb{R}^2$. A path is called *monotone* if any vertical or horizontal line intersects it in at most one connected component.

Fact 1. For any monotone path π between $p, q \in \mathbb{R}^2$, it holds that $|\pi| = |\overline{pq}|$.

Let P be a simple polygon with n vertices. We regard P as a compact set in $\mathbb{R}^2$, so its boundary ∂P is contained in P. An L_1 *shortest path* between p and q is a path joining p and q that lies in P and minimizes its L_1 length. The L_1 *geodesic distance* $d(p,q)$ is the L_1 length of an L_1 shortest path between p and q. We are interested in two quantities: the L_1 *geodesic diameter* $\operatorname{diam}(P)$ and *radius* $\operatorname{rad}(P)$ of P, defined to be $\operatorname{diam}(P) := \max_{p,q\in P} d(p,q)$ and $\operatorname{rad}(P) := \min_{p\in P} \max_{q\in P} d(p,q)$. Any pair of points $p, q \in P$ such that $d(p,q) = \operatorname{diam}(P)$ is called a *diametral pair*. A point $c \in P$ is said to be an L_1 *geodesic center* if and only if $\max_{q\in P} d(c,q) = \operatorname{rad}(P)$. We denote by $\operatorname{cen}(P)$ the set containing all L_1 geodesic centers of P.

Analogously, a path lying in P minimizing its *Euclidean* length is called the *Euclidean shortest path*. It is well known that there is always a unique Euclidean shortest path between any two points in a simple polygon [9]. We let $\pi_2(p,q)$ be the unique Euclidean shortest path from $p \in P$ to $q \in P$. The following states a crucial relation between Euclidean and L_1 shortest paths in a simple polygon.

Fact 2. (See Hershberger and Snoeyink [10].) For any two points $p, q \in P$, the Euclidean shortest path $\pi_2(p,q)$ is also an L_1 shortest path between p and q.

Notice that this does not imply coincidence between the Euclidean and the L_1 geodesic diameters or centers, as the lengths of paths are measured differently (see an example in Fig. 1). Nonetheless, Fact 2 enables us to exploit structures for Euclidean shortest paths such as the shortest path map.

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