Robotics and Computer-Integrated Manufacturing 27 (2011) 418–425

Contents lists available at ScienceDirect

Robotics and Computer-Integrated Manufacturing

journal homepage: www.elsevier.com/locate/rcim

Analysis of stiffness and elastic deformation of a 2(SP+SPR+SPU) serial–parallel manipulator

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ARTICLE INFO

Article history:
Received 12 November 2008
Received in revised form
10 September 2010
Accepted 13 September 2010

Keywords:
Serial–parallel manipulator
Kinematics
Statics
Stiffness matrix
Elastic deformation

ABSTRACT

A 2(SP+SPR+SPU) manipulator is a serial–parallel manipulator, which includes an upper manipulator and a lower manipulator. Its stiffness and elastic deformation are studied systematically in this paper. Firstly, a 2(SP+SPR+SPU) manipulator is constructed and its characteristics are analyzed. Secondly, the formulae for solving the elastic deformation and the compliance matrix of the active legs are derived and the elastic deformation and the total stiffness matrix of this manipulator are solved and analyzed. Finally, a finite element model of this manipulator is constructed and its elastic deformations are solved. The analytic solutions of elastic deformations of this manipulator are coincident with that of its finite element model.

1. Introduction

Serial manipulators have some merits such as larger workspaces, more flexibility, and a simple solution of forward kinematics [1], while parallel manipulators have some other merits such as higher stiffnesses, greater load-to-weight ratios, good stability, short transmission chains, and a simple solution of inverse kinematics [2,3]. In order to make up for their shortcomings, some parallel manipulators have been connected serially to form various serial–parallel manipulators (S–PMs), which are appropriate for multi-tasking machining such as milling, drilling, grinding, and numerical control machining. Stiffness is one of the most important performances of S–PMs, particularly when S–PMs are used as machine tools or robot arms/legs, because higher stiffness allows for higher machining speeds with higher accuracy of the end effector. Therefore, it is quite necessary to evaluate the stiffness and analyze the elastic deformation of S–PMs in the early design stage [4,5]. In the aspect of S–PMs, Romdhane [6] designed a hybrid serial–parallel Stewart like mechanism and analyzed its displacement kinematics. Tanev [7] analyzed the displacement kinematics of some hybrid (serial–parallel) robot manipulators. Using dual vectors and matrices, Sandipan and Ashitava [8] studied analytical determination of principal twists in serial, parallel, and hybrid manipulators. Based on two kinds of 3UPU manipulators, Zheng et al. [9] analyzed displacements of a hybrid S–PM. Lu and Leinonen [10] put forward a 2(3-RPS) S–PM and studied its displacement kinematics. They proposed a 2(3-SPR) S–PM and solved its active forces by CAD variation geometry [11] and also solved its velocity, acceleration, and statics by analytic approach and proved that the 2(3-SPR) S–PM has a much larger position and dexterous workspace than 2(3-RPS) S–PM [12]. Gallardo-Alvarado et al. [13,14] studied kinematics and dynamics of a 2(3-RPS) S–PM by the screw theory. Cha et al. [15] solved kinematic redundancy resolution of a S–PM by local optimization including joint constraints. Others designed or studied different S–PMs [16,17]. In the aspect of stiffness analysis, Huang et al. [18] estimated stiffness of a tripod-based parallel manipulator (PM) by decomposing the whole machine structure into two separate substructures. Similarly, Zhang and Lang Sherman [19] established a stiffness modeling for a limited-DOF parallel manipulator (PMs). Ceccarelli and Carbone [20] analyzed stiffness of cassino PM in view of the motions of every joint and link. Carbone and Ceccarelli [21] deduced the stiffness matrix of a hybrid parallel–serial manipulator. Li and Xu [22–24] studied kinematics of a 3-PRS PM and analyzed the mobility and stiffness of a 3-PUU PM and a 3-PRC PM. Others analyzed stiffness of some other PMs [25–27]. Since some 3-DOF PMs have the coupled structure constraints, which may result in an unnecessary tiny self-motion [28,29], when a serial–parallel manipulator is created by these 3-DOF PMs, the unnecessary tiny self-motion of the end effector may be enlarged. Thus, applications of the S–PMs are limited.

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doi:10.1016/j.rcim.2010.09.002

Lu and Hu [30] studied kinematics and statics of a 2(SP+SPR+SPU) manipulator, in which the unnecessary tiny self-motion is removed effectively by a SP-type active constrained leg. However, no studies have been found in the stiffness and elastic deformation of the 2(SP+SPR+SPU) manipulator. Therefore, this article focuses on the analysis of the stiffness and the elastic deformation of the 2(SP+SPR+SPU) manipulator because it has large position/dexterous workspace and some potential applications for the robot arm, leg, and twist, the machine tools, sensor, surgical manipulator, tunnel borer, the barbette of war ship, and satellite surveillance platform and so on.

2. Characteristics of 2(SP+SPR+SPU) manipulator

A 2(SP+SPR+SPU) manipulator is a serial–parallel manipulator with 6 DOFs [30], as shown in Fig. 1.

It is composed of a lower SP+SPR+SPU PM and an upper SP+SPR+SPU PM. The two PMs are connected serially, thus the workspace and the flexibility of the 2(SP+SPR+SPU) manipulator are enlarged obviously. The lower PM is composed of a middle moving platform m, a fixed base B, one spherical joint-active prismatic joint (SP) active leg r_1 with a linear actuator, one spherical joint-active prismatic joint-revolute joint (SPR) active leg r_2 with a linear actuator, and one spherical joint-active prismatic joint-universal joint (SPU) active leg r_3 with linear actuator. Here, m is a regular triangle with 3 vertices (b_1, b_3, b_3), 3 sides $l_i = l$ and a central point o; B is a regular triangle with 3 vertices (B_1, B_2, B_3), 3 sides $L_i = L$ and a central point O. Let $\{m\}$ be a coordinate frame o-xyz fixed on m at o, $\{B\}$ be a coordinate frame O-XYZ fixed on B at O, $\perp$ be a perpendicular constraint, and $||$ be a parallel constraint. The SP leg r_1 connects B with m by a spherical joint S on B at B_1, an active prismatic joint P fixed on m at b_1. The SPR leg r_2 connects B with m by S on B at B_2, P, and a revolute joint R on m at b_2. The SPU active leg r_3 connects B with m by S on B at B_3, an active P, and U on m at b_3. Some constraints ($r_1 \perp m$, $z \perp m$, $x \perp r_1$, $y \perp r_1$, $x \perp b_1b_2$, $y || b_1b_2$, $x || R$, $r_2 \perp R$, $Z \perp B$, $X \perp B_1B_2$, and $Y || B_1B_2$) are satisfied.

The upper manipulator is similar to the lower manipulator, except that m is replaced by m_1 with a central point o_1 and 3 vertices (b_{11}, b_{21}, b_{31}); the base B is replaced by a platform c with a central point o and 3 vertices (B_{11}, B_{21}, B_{31}); r_i and R are replaced by r_{i1} and R_1, respectively. Let $\{c\}$ be a coordinate frame o-$x_cy_cz_c$ fixed on m at o. Let $\{m_1\}$ be a coordinate frame o_1-$x_1y_1z_1$ fixed on m_1 at o_1. Some constraints (c and m being coplanar, z and z_c being collinear, and an angle between y and y_c being $\theta = 60°$) are satisfied. Since the coupled structure constraints can be transformed into the decoupled structure constraints by the SP constrained active leg, the unnecessary tiny self-motion motions can be removed effectively. In addition, because most joints are spherical or prismatic joints, it is simple in structure.

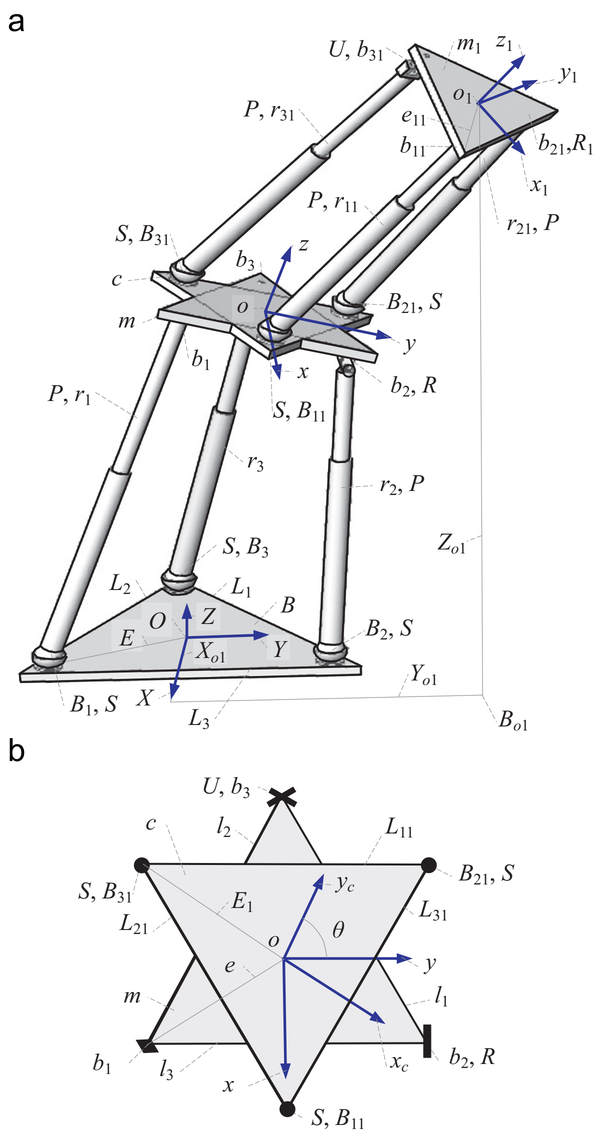

Fig. 1. A 2(SP+SPR+SPU) manipulator (a) and its composite platform (b).

3. Kinematics/statics of 2(SP+SPR+SPU) manipulator

A lower PM with kinematic parameters and its force situation are shown in Fig. 2.

The whole workloads can be simplified as a wrench ($\boldsymbol{F}$, $\boldsymbol{T}$) applied onto platform m at o. $\boldsymbol{F}$ is a concentrated force and $\boldsymbol{T}$ is a concentrated torque. ($\boldsymbol{F}$, $\boldsymbol{T}$) includes the inertia wrench and the gravity of m, and inertia wrench and the gravity of the active legs, which can be mapped into a part of the whole workload and the external working wrench (such as machining or operating wrench of tool and damping wrench of end effector). ($\boldsymbol{F}$, $\boldsymbol{T}$) are balanced by 3 active forces $\boldsymbol{F}_{ai}$ ($i = 1, 2, 3$) and 3 constrained forces $\boldsymbol{F}_{ci}$. Here, $\boldsymbol{F}_{ai}$ is applied on and along r_i at B_i, its unit vector $\boldsymbol{\delta}_i$ is the same as that of r_i. After velocity/acceleration of platform of PM are solved, the dynamic ($\boldsymbol{F}$, $\boldsymbol{T}$) can be solved [31].

In the lower PM, the SP constrained active leg r_1 and the SPR constrained active leg r_2 should be transformed into the

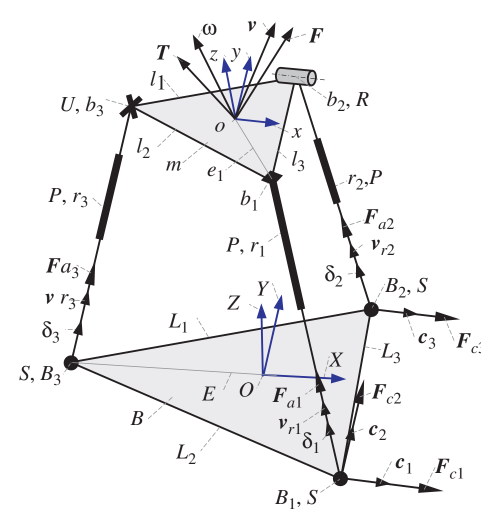

Fig. 2. Lower SP+SPR+SPU PM with kinematics parameters and it force situation.