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Connectivity with directional antennas in the symmetric communication model ☆

S. Dobrev [a], M. Eftekhari [b], F. MacQuarrie [c], J. Maňuch [d], O. Morales Ponce [e], L. Narayanan [b], J. Opatrny [b,*], L. Stacho [d]

[a] Inst. of Mathematics, Slovak Academy of Sciences, Slovakia
[b] Dept. of Comp. Science and Software Eng., Concordia University, Canada
[c] School of Comp. Science, Carleton University, Canada
[d] Dept. of Comp. Science, University of British Columbia, Canada
[e] Dept. of Eng. and Comp. Science, California State University, Long Beach, USA

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ABSTRACT

We study the problem of connectivity in wireless networks in which each node uses a single directional antenna. We consider the symmetric model of communication with directional antennas. In this model, two nodes are connected by a link if and only if they lie in the transmission sectors of each other's antenna. Given nodes located in the plane, each with an antenna of beamwidth ϕ and radius r, we study the problem of orienting the antennas in such a way as to create a connected network. We show that for $\phi < \pi/3$ and a given radius, the problem of determining the existence of an orientation that ensures a connected network is NP-complete. For $\phi \geq \pi/2$, in which case connectivity is known to be always possible, we study the problem of achieving connectivity while minimizing the radius. For different ranges of ϕ, we give approximation algorithms that achieve connectivity while using a radius that is at most a constant times the optimal radius. Some of our algorithms also have provable bounds on the stretch factor of the resulting network compared to a network of nodes with omnidirectional antennas of a given radius.

1. Introduction

Wireless networks have traditionally employed nodes with omnidirectional antennas to transmit and receive data using wireless transmission. The transmission and reception areas of such antennas can be modeled as a circular area around the antenna, where the radius of the circle depends on the power used by the transmitter. Recent years have seen great advances in the development of directional antennas for practical use in wireless networks [1]. A directional antenna employs a directional beam of a fixed angle, and is able to reach much greater distances for the same amount of transmitter power.

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* Corresponding author at: Dept. of CSE, Concordia University, 1455 deMaisonneuve o. Montreal, H3G 1M8 Canada. Tel.: +1 514 848 2424 x3047.
E-mail addresses: Stefan.Dobrev@savba.sk (S. Dobrev), m_eftek@encs.concordia.ca (M. Eftekhari), frasermacquarrie@gmail.com (F. MacQuarrie), jan.manuch@gmail.com (J. Maňuch), oscarmponce@gmail.com (O. Morales Ponce), lata@cs.concordia.ca (L. Narayanan), opatrny@cs.concordia.ca (J. Opatrny), ladislav_stacho@sfu.ca (L. Stacho).

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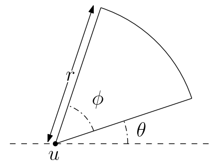

Fig. 1. Node with a directional antenna of radius r and beamwidth ϕ with orientation θ.

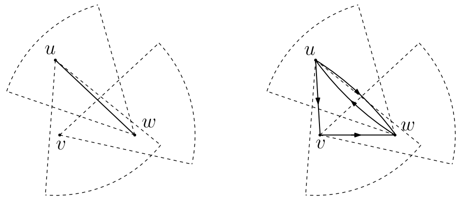

Fig. 2. Symmetric model (left) and asymmetric model (right). Solid lines represent communication links.

This indicates that large energy savings may be possible in a network with the use of directional antennas [1]. The use of directional beams can also reduce interference with neighboring nodes.

The transmission area of a directional antenna can be modeled as a *sector* of fixed angle ϕ and radius r subtended at the wireless node (see Fig. 1). We call ϕ the *beamwidth* of the antenna, and r its *range*. As in the omnidirectional case, the range of the antenna is a function of the power used by the transmitter. We assume that the antenna can be assigned a specific *orientation* θ at every node to ensure connectivity of the network. Two models have been proposed for communication between wireless nodes using directional antennas. In the first model, it is assumed that a node sends signals only to nodes within the sector of its directional antenna, but it can receive signals from all directions. Thus a node u with a directional antenna of beamwidth ϕ can transmit to any node v that lies in the sector of radius r and angle ϕ subtended at u. Notice that v's beam may be pointed away from u and so u may not lie in v's sector. Thus it is possible that u can transmit to v but v cannot transmit to u. This model has been studied widely (see [2] for a survey), and we call it the *asymmetric model*. In the asymmetric model, communication links are directed links, and to ensure communication between a set of wireless nodes, it is required that the communication graph induced by the directional antennas is strongly connected. See Fig. 2 for an illustration.

In the second model, which we call the *symmetric model*, node u can communicate with node v if and only if v lies in the sector of u and u lies in the sector of v [3]. Thus each node sends and receives only from within the sector of its antenna. We call such an antenna a *DD antenna* (Directional transmission, Directional reception). Communication links in networks with DD-antennas are always symmetric, and the communication graph of such a network is required to be connected to ensure communication in the network. See Fig. 2 for an illustration. In this paper, we assume the symmetric model of communication. Observe that whether or not two nodes located in the plane are connected is a function of three factors: the beamwidth, range, and orientation of their respective antennas. We assume that each node is equipped with a DD-antenna, and that antennas of all nodes have the same beamwidth and range. However the orientation of antennas can vary among the nodes. Given a set of nodes in the plane, each with an antenna of a given beamwidth ϕ, we are interested in orienting the antennas so that the resulting network is connected. It is known that for beamwidth $\phi \geq \pi/3$, such an orientation is always possible, while for beamwidth $< \pi/3$ it may not be possible [3]. The first question of interest, therefore, is the complexity of determining the existence of an orientation to ensure connectivity for $\phi < \pi/3$. Since a smaller antenna range results in less energy consumption, we proceed to study the problem of *minimizing the range* that ensures the connectivity. Finally, we study the trade-offs involved in replacing omnidirectional antennas in a given network with directional antennas:

(a) By how much do we need to increase the range relative to the omnidirectional antennas, and what is the stretch factor of the resulting graph?
(b) Given a connected unit disk graph that represents a network of nodes with omnidirectional antennas, we are interested in replacing the omnidirectional antennas with DD antennas while minimizing the range and ensuring that the communication graph of the directional antennas is a c-hop spanner of the original unit disk graph, that is a spanning subgraph with the property that any edge in the original graph corresponds to a path of at most c edges in the subgraph.

1.1. Related work

The problem of connectivity in wireless networks with directional antennas was first studied by Caragiannis et al. in [4]. The authors proposed the asymmetric model of communication, where nodes have directional transmitters but omnidirectional receivers. They proved the NP-completeness of determining connectivity when the antenna beamwidth is less than $2\pi/3$, and presented algorithms to approximate or find the minimum transmission range for connectivity for larger antenna