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On k-greedy routing algorithms

Huaming Zhang [a,*], Xiang-Zhi Kong [b]

[a] Computer Science Department, The University of Alabama in Huntsville, Huntsville, AL 35899, United States
[b] School of Science, Jiangnan University, Wuxi, 214122, China

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ABSTRACT

Let $G = (V, E)$ and $G' = (V', E')$ be two graphs. A *k-inverse-adjacency-preserving mapping* ρ from G to G' is a one-to-many and onto mapping from V to V' satisfying the following: (1) Each vertex $v \in V$ in G is mapped to a non-empty subset $\rho(v) \subset V'$ in G', the cardinality of $\rho(v)$ is at most k; (2) if $u \neq v$, then $\rho(u) \cap \rho(v) = \emptyset$; and (3) for any $u' \in \rho(u)$ and $v' \in \rho(v)$, if $(u', v') \in E'$, then $(u, v) \in E$. A vertex u' in $\rho(u)$ is called a *virtual location* for u.

Let ρ be a k-inverse-adjacency-preserving-mapping (k-IAPM for short) from G to G'. Let δ be a greedy drawing of G' into a metric space $\mathcal{M}$. Consider a message from u to be delivered to v in G. Using the k-IAPM ρ from G to G', intuitively, one can treat the message to be routed as if it were from one virtual location u' for u to one virtual location v' for v, except that all the virtual locations of a vertex u were identified with each other (which can be thought as instantaneously synchronized mirror sites for a particular website, for example). Then a routing path P' can be computed from u' to v' while identifying all virtual locations for any vertex in G. Since ρ inversely preserves adjacency from G' to G, such a routing path P' corresponds to exactly one routing path P in G, which connects u to v.

In this paper, we formalize the above intuition into a concept which we call k-greedy routing algorithm for a graph G with n vertices, where k refers to the maximum number of virtual locations any vertex of G can have. Using this concept, the result presented in [18] can be rephrased as a 3-greedy routing algorithm for 3-connected plane graphs, where the virtual coordinates used are from 1 to $2n-2$. In this paper, we present a 2-greedy routing algorithm for 3-connected plane graphs, where each vertex uses at least one but at most two virtual locations numbered from 1 to $2n-1$. For the special case of plane triangulations (in a plane triangulation, every face is a triangle, including the exterior face), the numbers used are further reduced to from 1 to $\lfloor \frac{5n+1}{3} \rfloor$. Hence, there are at least $\lceil \frac{n-1}{3} \rceil$ vertices that use only one virtual location.

1. Introduction

Routing is one of the most important algorithmic problems in networking. Extensive research has been devoted to discover efficient routing algorithms (see [4,13,17]). Routing was primarily done and still is mainly done via routing protocols (e.g., see [4,13,17]). This approach is space inefficient and requires considerable setup overhead, which makes it infeasible

* Corresponding author.
E-mail addresses: hzhang@cs.uah.edu (H. Zhang), xiangzhikong@jiangnan.edu.cn (X.-Z. Kong).

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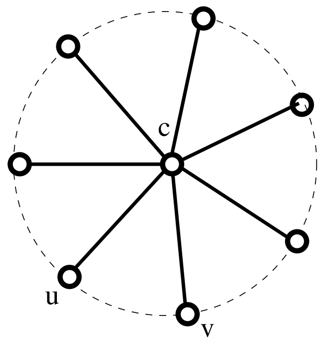

Fig. 1. A star graph $K_{1,7}$. u is at a void position.

for some networks (e.g., wireless sensor networks). Recently, an alternative approach *geometric routing* has been proposed. Geometric routing uses geometric coordinates of the vertices to compute the routing paths. The simplest geometric routing is *greedy routing*, in which a vertex simply forwards messages to a neighbor that is closer to the destination than itself. Greedy routing algorithms are conceptually simple and they only rely on local coordinates information to compute the routing paths. However, they cannot always guarantee message delivery. For example, for a star-shaped network $K_{1,7}$ embedded in $\mathcal{R}^2$ in Fig. 1, greedy routing fails due to the fact that the node u is at a *void position*: all neighbors (only one) of u are farther from the destination v than itself in the embedding. As a matter of fact, $K_{1,7}$ does not admit any greedy drawing at all in $\mathcal{R}^2$.

Many solutions have been sought to solve the delivery problem. In particular, an elegant solution was proposed by Rao et al. in [15]. Instead of using the real geometric coordinates (e.g., GPS coordinates), one could use graph drawing, based on the structure of a network G, to compute vertex coordinates in the drawing. The drawing coordinates are used as the *virtual coordinates* of the vertices of G. Then geometric routing algorithms rely on virtual coordinates to compute routes. Simply speaking, a *greedy drawing* is a drawing in which greedy routing works. More precisely:

Definition 1. (See [16].) Let $\mathcal{M}$ be a set. A metric function $d(*,*)$ of $\mathcal{M}$ is a function $d : \mathcal{M} \times \mathcal{M} \to \mathcal{R}$ that satisfies the following four conditions: (1) $d(x, y) \geq 0$, for any $x, y \in \mathcal{M}$; (2) $d(x, y) = 0$ if and only if $x = y$, for any $x, y \in \mathcal{M}$; (3) $d(x, y) = d(y, x)$; and (4) $d(x, y) \leq d(x, z) + d(z, y)$, for any $x, y, z \in \mathcal{M}$.

Let $\mathcal{M}$ be a set and $d(*,*)$ be a metric function of $\mathcal{M}$. Let $G = (V, E)$ be a graph.

1. A *drawing* of G into $\mathcal{M}$ is a mapping $\delta : V \to \mathcal{M}$ such that $u \neq v$ implies $\delta(u) \neq \delta(v)$.
2. A drawing δ is a *greedy drawing* with respect to d if for any two vertices u, v of G ($u \neq v$), u has a neighbor w such that $d(\delta(u), \delta(v)) > d(\delta(w), \delta(v))$.

Consider any two vertices $u \neq v$ of G in the greedy drawing δ. According to the definition of a greedy drawing, it is easy to see that there is a *virtual distance decreasing path* between u and v in the drawing with respect to the metric function d_δ (see [16]). Namely, there is a path $(u = v_1, v_2, \cdots, v_k = v)$ in G such that $d_\delta(v_i, v) > d_\delta(v_{i+1}, v)$, for each i from 1 to $k - 1$. Therefore, greedy routing simply forwards the message from u to a neighbor w, which is closer to the destination v than u. The forwarding process continues and the distance to the destination v keeps dropping. Eventually, the distance becomes 0 and the message reaches the destination v. Therefore, when there is a greedy drawing of G into a metric space $\mathcal{M}$, greedy routing always succeeds.

Later, intensive research focused on finding greedy routing algorithms via greedy drawings of certain category of graphs into certain metric spaces. The graphs under consideration are mostly 3-connected plane graphs. The metric spaces used were $\mathcal{R}^2$, $\mathcal{R}^3$, and hyperbolic plane, etc. For more information on various greedy routing algorithms via greedy drawings, we refer readers to [1,5,6,8,11,14–16,18].

Zhang et al. observed that greedy routing can be generalized to *semi-metric spaces* [18]. Instead of using greedy drawing of graphs into metric spaces, they used *greedy embedding* of graphs into *semi-metric spaces* for greedy routing purposes. We have the following from [18]:

Definition 2. (See [18].) Let S be a set. A semi-metric of S is a function $d : S \times S \to \mathcal{R}$ that satisfies the following three conditions: (1) $d(x, y) \geq 0$, for any $x, y \in S$; (2) $d(x, y) = 0$ if and only if $x = y$, for any $x, y \in S$; and (3) $d(x, y) = d(y, x)$, for any $x, y \in S$.

Let S be a set and $d(*,*)$ a semi-metric function of S. Let $G = (V, E)$ be a graph.

1. An *embedding* of G into S is a mapping $\delta : V \to S$ such that $u \neq v$ implies $\delta(u) \neq \delta(v)$.
2. An embedding δ is a *greedy embedding* with respect to d if for any two vertices u, v of G ($u \neq v$), u has a neighbor w such that $d(\delta(u), \delta(v)) > d(\delta(w), \delta(v))$.