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Architecture optimisation of three 3-PRS variants for parallel kinematic machining

Geoffrey Pond, Juan A. Carretero*

Department of Mechanical Engineering, University of New Brunswick, Fredericton, NB, E3B 5A3 Canada

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Abstract

The 3-PRS parallel manipulator has recently been prototyped as a machining centre. PRS denotes the prismatic–revolute–spherical architecture of each limb where only the prismatic joint is actuated and hence underlined. Three variations of this manipulator have been independently presented in the literature. The architectural parameters affecting the size of the dexterous workspace volume are identified for each of the three models. The influence of these parameters on each of the three models is studied. Next, the manipulators' dexterity is defined as the architecture's ability to lend stiffness and accuracy to the machine tool.

By studying the singular values and condition number of a newly developed square, dimensionally homogeneous Jacobian matrix, regions of the workspace corresponding to capabilities for high end effector (EE) velocities in each of the degrees of freedom (DOF) are identified. Optimisation of the architectural parameters is then completed to provide the largest possible size for this region.

The dexterous workspace size, obtained using optimal architectural parameters unique to each variant, is then compared between each of the three variants of the 3-PRS manipulator.

Keywords: 3-PRS manipulator; Singularity analysis; Dexterity; Workspace; Optimisation

1. Introduction

Parallel manipulators have recently experienced more widespread attention as their various advantages become better known. They have been successfully implemented in applications where advantages such as high stiffness, potentially higher end effector (EE) velocities, and an ability to handle higher payloads, are of great importance. One such application is that of parallel kinematic machines (PKMs). Both the Variax machining centre, produced by Giddings & Lewis and Fanuc's F-200i are based on the Stewart–Gough architecture [1]. In fact, a variety of other machining centres employing parallel kinematic architectures are available on the market from Ingersoll, Honda, and Okuma to name only a few.

During the initial design phase of a PKM, or for any other robotic application for that matter, the architectural parameters of the manipulator may be optimised to yield an architecture having desired design characteristics. Dexterity is an important such characteristic. It describes the sensitivity of the EE, or spindle, to changes in the actuated joints, at a specific pose. This sensitivity indicates relative stiffness of the architecture at that pose, or the relative speed at which the manipulator may alter the spindle's position and/or orientation.

This paper will focus on the study and optimisation of the dexterous workspace volume for the 3-PRS parallel manipulator first described in [2]. This manipulator has been studied as a machining centre in [3]. Dexterity analysis of this manipulator presents a formidable challenge as conventional methods used to measure the dexterity of such a mechanism are not necessarily viable, as will be explained in the following section.

1.1. Dexterity

Dexterity is often defined qualitatively as the ease by which a manipulator may move. This vague definition has

*Corresponding author. Tel.: +1 506 458 7454; fax: +1 506 453 5025.
E-mail address: Juan.Carretero@unb.ca (J.A. Carretero).

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left many researchers to apply their own mathematical method in which to measure dexterity.

When considered as the sensitivity of the EE pose for actuator displacements, dexterity may be used as an important design characteristic of machining centre architectures. More specifically, a proposed architecture may be optimised to yield increased stiffness and accuracy. Several works have used the singular values and/or the condition number of the Jacobian matrix as a dexterity index [4–6]. The Jacobian matrix, $\mathbf{J}$, relates the EE velocity vector $\dot{\mathbf{x}}$ to the actuator velocity vector $\dot{\mathbf{q}}$ by

$$\dot{\mathbf{q}} = \mathbf{J}\dot{\mathbf{x}}. \tag{1}$$

Physically, the condition number of $\mathbf{J}$, here denoted by κ, may be interpreted as the ratio of the magnitude of vector $\dot{\mathbf{q}}$ required to move the EE in its fastest direction to that required to move the EE in its slowest direction. Like $\mathbf{J}$, κ is dependent on the pose of the manipulator. The condition number is a function of the singular values of a matrix where the number of singular values corresponds to the number of DOF for the system, or the rank of $\mathbf{J}$ when the manipulator is not at a singular pose [7]. The magnitude of the actuators' effort ($|\dot{\mathbf{q}}|$) required to move in the 'slowest direction' (i.e., direction with the best resolution) is represented by the maximum singular value, $\sigma_{\max}$.

It is well known that architectural stiffness is an important characteristic of machining centres, not only for the accuracy of the workpiece, but in order to avoid undesired vibration of the machine tool which may prelude chatter. Large singular values suggest that the architecture is relatively stiff under loading of the machine tool at that specific pose.

Conversely, the minimum singular value $\sigma_{\min}$ corresponds to the actuators' effort required to move in the 'fastest direction'. Clearly, at such poses, the architecture is able to alter the spindle position and orientation with relatively little effort by the actuators. In machining applications, this is typically secondary in importance to accuracy in stiffness.

The condition number may be expressed mathematically as

$$\kappa = \frac{\sigma_{\max}}{\sigma_{\min}}. \tag{2}$$

Therefore, for a manipulator pose where constant actuator effort $|\dot{\mathbf{q}}|$ is required, regardless of the direction of the unit vector $\dot{\mathbf{x}}$, the Jacobian matrix condition number is equal to 1. Such poses are termed isotropic configurations [8].

As popular as this measure has been, it still suffers from two major drawbacks. First, the measure is relative. That is, if the manipulator is able to move equally fast in any direction, i.e., $|\dot{\mathbf{q}}|$ is constant and relatively small for any $\dot{\mathbf{x}}$ such that $|\dot{\mathbf{x}}| = 1$, the Jacobian matrix condition number is one. This result is the same for a pose where the manipulator moves equally slow in any direction, i.e., $|\dot{\mathbf{q}}|$ is constant and relatively large for any $\dot{\mathbf{x}}$ such that $|\dot{\mathbf{x}}| = 1$. Therefore, this method is not able to distinguish between these very different areas of the workspace, thus defeating the purpose of pursuing detailed analyses of dexterous characteristics of manipulators.

Second, due to dimensional dependencies within the conventionally defined Jacobian matrix, this method should be restricted to (a) manipulators where only one type of actuator is used (either revolute or prismatic, but not a combination of both) and (b) where only either rotational or translation DOF are obtained by the mechanism (but not a combination of both). The first restriction is not profound as most parallel manipulators to date have only one type of actuation. However, the vast majority of manipulators have both translation and rotational DOF such as 5-axes PKMs. The only exceptions being pure rotational and pure translational manipulators such as the Agile Eye [9] and the Delta manipulator [10] to name a few.

Characteristic of 5-axes PKMs, the defined DOF for the 3-PRS manipulator analysed here include both translational and rotational motions. Therefore, an alternative method must be used to circumvent the barriers described in the previous paragraph.

1.2. Jacobian formulation

As mentioned earlier, the restrictions on the use of the Jacobian matrix condition number for measuring dexterity are the result of dimensional inconsistencies that may exist within the elements of the Jacobian matrix itself. To avoid this, Kim and Ryu [11] have presented a method, based on the work by Gosselin [12], to develop dimensionally homogeneous Jacobian matrices. Unfortunately, the meaning of the singular values of such non-square Jacobian matrices is obscure and therefore the use of the singular values in finding the matrix condition number is unadvisable. This is due to the fact that the Jacobian matrix presented by Kim and Ryu [11] contains nine columns corresponding to a set of nine variables used to represent the EE velocity. Of these nine, at most n are independent, where n represents the number of DOF of the manipulator. The eigenvectors of $\mathbf{J}^{\mathrm{T}}\mathbf{J}$ could therefore conceivably correspond to directions within the task space which are not viable.

Pond and Carretero [13] have furthered the work introduced in [11], to develop a square, dimensionally homogeneous Jacobian matrix whose singular values have apparent meaning. These singular values may then be used to measure dexterity of a mechanism via calculation of the square, dimensionally homogeneous Jacobian matrix condition number and by examination of the magnitude of the singular values. Since this method will be used in later sections to measure the performance of the 3-PRS manipulator, a brief review is presented here. Note that the work in [13] is similar in concept to that presented in [14] where the former is analytical and the latter is a numerical approximation.