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Curvature based shape detection

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A R T I C L E I N F O

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A B S T R A C T

In this paper is defined a notion of discrete curvature associated with a discrete piecewise-smooth curve. Since every planar curve is, up to the orientation preserving isometry, uniquely determined by its curvature, shape recognition is reduced to fitting estimated discrete curvature function. Based on shape attributes, measures of similarity between two objects are introduced and used for full classification of detected geometric shapes in binary edge image.

1. Introduction

The problem of detecting geometric shapes such as lines, circles, circular arcs, ellipses etc. is a central problem in pattern recognition, computer vision, and robotics. Our ultimate goal in this direction is recognition of arbitrary geometric shapes.

In this paper we assume that the given raster image is black and white, i.e. the shapes are black pixels on white background. We define a notion of discrete curvature and apply it to the detection of piecewise-smooth curves.

The paper is organized as follows. In Section 2 we present related approaches addressing shape recognition problem. In Section 3 we recall some basic definitions and properties of plane curves.

In Section 4 we define a notion of discrete curvature associated with black pixel. It is analogous to the curvature of smooth plane curve. The curvature is a local property of smooth curve: it depends only on the shape of the curve near the feature point. Similarly, we use only the neighboring pixels to calculate the discrete curvature of the feature black pixel.

The process of assigning discrete curvature function to the discrete curve is presented in Section 5. We explain in detail how we trace points along the curve and calculate the corresponding curvature.

Section 6 is dedicated to the methods of discrete curvature matching and object recognition. Some measures of similarity between two objects based on their shape attributes are introduced and later used for distinguishing objects of different types. In addition, we can assess the attributes of similarity for objects from the same class, i.e. scaling factors and angles of rotation.

We discuss error estimations in Section 7. Calculation errors originate from both the discrete nature of image space and the numerical computations and we investigate the impact of each one on the outcome of the detection process.

Finally, some experimental results and conclusions are given in Section 8.

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2. Related work

Hough Transform (HT) is one of the most widely used algorithms for line detection [1,2]. General Hough Transform (GHT) represents a modification of HT which is used to detect circles and even arbitrary curves [3]. The idea is that each black pixel in the image space votes in parameter space and the votes are accumulated in Hough space (HS) of parameters.

GHT is a very robust algorithm but it has some serious issues. The main problem in line detection using HT is line thickness, but recent work solves that problem efficiently [4].

Drawbacks of GHT for curve detection include high memory requirements, line thickness, and local maximum detection in the parameter space. Use of GHT for detection of more complex shapes is very limited, because of the large number of parameters which require high amounts of memory. Ellipse, for example, is determined by five parameters in general case (coordinates of center, size of two axes and rotation), hence, in principle, requires a 5-dimensional parameter array. The dimension of parameter space can be reduced by using some additional information [3,5], but the computational load is still high and some other difficulties arise, especially in arc detection process.

Besides Hough-based methods for shape recognition, there are several different approaches for solving this problem.

The first approach consists of converting the object to a set of idealized thin lines called the skeleton or medial axis. In this way the thinnest representation of the original object that preserves the homotopy aiding synthesis and understanding is obtained. In order to get structural description that captures the topological information embedded in the skeleton, one must detect end points, junction points and curve points of medial axis. The thin lines can be converted into a graph associating the curve points with the edges, the end and the junction points with the vertices. Such a skeletal graph can then be used as an input to graph matching algorithms (see [6–9]).

Shapes have also been represented by their outline curves. Matching typically involves finding a mapping from one curve to the other that minimizes an "elastic" performance functional, which penalizes "stretching" and "bending" [10,11]. The curve-based methods in general suffer from one or more of the following drawbacks: asymmetric treatment of the two curves, lack of rotation and scaling invariance, and sensitivity to articulations and deformations of parts.

The type of representation used in describing a shape can have a significant impact on the effectiveness of the recognition strategy. The comparative analysis of different shape representations can be found in [12].

Other approaches include approximation by special types of curves such as B-spline curves [13], Bézier curves [14], Hilbert curve [15], etc.

Previous approaches are mostly applied to general shape detection. For the detection of parameterized curves, the generalized Radon transform could be used [16].

The idea of curvature based shape representation for planar curves was introduced in [17,18]. There are many methods for estimating the curvature at point P (see [19–22]), majority of them use approximation and curvature fitting. Invariant descriptors under the affine, similarity and projective transformations were studied in [23]. Here, we use the similar approach.

3. Curvature of planar curves

Consider a parameterized curve $\alpha : (a, b) \to \mathbb{R}^2$ parameterized by its arc length s, i.e. distance along the curve measured from some fixed point on the curve:

$$s(t) = \int_c^t \left\| \alpha'(u) \right\| du, \quad c \le t \le b.$$

Then $\alpha'(s)$ is its tangent vector $T(s)$ and $\alpha''(s)$ is collinear to the unit normal vector $N(s)$. Therefore we have

$$\alpha''(s) = k(s)N(s), \tag{1}$$

where the function $k(s)$ is called **curvature** of the curve α at point $\alpha(s)$.

Our opinion is that the curvature approach can be used for detecting arbitrary shapes due to the fundamental theorem for plane curves (for mathematical details see [24]). The importance of the theorem is that the shape of the curve is determined only by its (signed) curvature, regardless of the position of the curve in the plane, i.e. translation and rotation.

To address the problem of scaling (i.e. detecting the same shape of different size) we suggest the following. Suppose that curve α_λ is obtained from curve $\alpha = \alpha(s)$, $s \in (0, b)$ by scaling with coefficient λ. Then the curvature of α_λ is scaled by $\frac{1}{\lambda}$ and arc length is scaled by λ, so we have

$$\bar{k}(\lambda s) = \frac{1}{\lambda} k(s), \quad s \in (0, b), \tag{2}$$

where $\bar{k}$ denotes the signed curvature of the curve α_λ. This allows us to easily match scaled shapes (details are found in Section 6).