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Capture bounds for visibility-based pursuit evasion ☆

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ABSTRACT

We investigate the following problem in the visibility-based discrete-time model of pursuit evasion in the plane: *how many pursuers are needed to capture an evader in a polygonal environment with obstacles under the minimalist assumption that pursuers and the evader have the same maximum speed?* When the environment is a simply-connected (hole-free) polygon of n vertices, we show that $\Theta(n^{1/2})$ pursuers are both necessary and sufficient in the worst-case. When the environment is a polygon with holes, we prove a lower bound of $\Omega(n^{2/3})$ and an upper bound of $O(n^{5/6})$ for the number of pursuers that are needed in the worst-case, where n is the total number of vertices including the hole boundaries. More precisely, if the polygon contains h holes, our upper bound is $O(n^{1/2}h^{1/4})$, for $h \leq n^{2/3}$, and $O(n^{1/3}h^{1/2})$ otherwise. We then show that with additional assumptions these bounds can be drastically improved. Namely, if the players' movement speed is small compared to the "feature size" of the environment, we give a deterministic algorithm with a worst-case upper bound of $O(\log n)$ pursuers for simply-connected n-gons and $O(\sqrt{h} + \log n)$ for multiply-connected polygons with h holes. Further, if the pursuers are allowed to randomize their strategy, regardless of the players' movement speed, we show that $O(1)$ pursuers can capture the evader in a simply connected n-gon and $O(\sqrt{h})$ when there are h holes with high probability.

1. Introduction

In visibility-based pursuit-evasion, a team of *pursuers* is tasked with locating and capturing an adversary, called the *evader*, within a polygonal environment. The pursuers are equipped with cameras, able to maintain omni-directional line-of-sight visibility, and must plan and coordinate their moves until some pursuer can reach the same location as the evader. The problem is motivated by applications in robotics, and has drawn a significant interest since it was introduced by Suzuki and Yamashita [24], although much of the prior work has focused on the simpler problem of *evader detection*, where the pursuers win as soon as the evader is "seen" by some pursuer [10–12,20,25].

In this paper, we consider the complexity of physically capturing the evader within this visibility-based framework of pursuit-evasion. For our main results, we make only the minimally necessary assumption that all players (pursuers and evader) have equal maximum speed, *which is normalized to one* by appropriate scaling of the environment. On its turn, each

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player can move along a path to any position within distance one of its current location, where the distance is measured using the shortest path (geodesic) distance avoiding the obstacles in the environment. The game is played in the discrete, alternating turn model: the evader moves first, and then the pursuers. (One practical reason for adopting the discrete time model is that the differential equations modeling the continuous time game are intractable for all but the simplest of environments.)

Our first result gives a tight bound of $\Theta(n^{1/2})$ for the number of pursuers needed to capture the evader when the environment is a simply-connected (hole-free) polygon of n vertices. Generalizing this result, we show that at least $\Omega(n^{2/3})$ pursuers are needed for capture in polygons with holes. Complementing this lower bound, we prove an upper bound of $O(n^{1/2}h^{1/4})$, for $h \leq n^{2/3}$, and $O(n^{1/3}h^{1/2})$ otherwise, where h is the number of holes in the polygon. More simply, the upper bound is $O(n^{5/6})$. We then show with additional assumptions these bounds can be drastically improved. Namely, if the players' movement speed is small compared to the "features size" of the environment, we give a deterministic algorithm with a worst-case upper bound of $O(\log n)$ pursuers for simply-connected n-gons and $O(\sqrt{h} + \log n)$ for multiply-connected polygons with h holes. Further, if the pursuers are allowed to randomize their strategy, regardless of the players' movement speed, we show that $O(1)$ pursuers can capture the evader in a simply connected n-gon and $O(\sqrt{h})$ when there are h holes with high probability.

Related work The history of pursuit-evasion games in geometric environments is long, and can be traced to the celebrated "Lion-and-Man" problem, attributed to Rado in 1930s: if a man and a lion are confined to a closed arena, and both have equal maximum speeds, can the lion catch the man? Surprisingly, the man can evade the lion indefinitely as shown by Besicovitch [18]—the lion fails to reach the man in any finite time although it can get arbitrarily close to him [2]. Extending this result to environments with obstacles, however, has proved difficult, and the only relevant result seems to be a recent work of Karnad and Isler [13] that deals with a single circular obstacle! Pursuit evasion is also studied as a form of differential games and solved using the Hamilton–Jacobi–Isaacs equation. Unfortunately, the resulting system of differential equations is intractable for all but the simplest of the environments, and unsuited for the complex, multiply-connected environments we study in this paper. An interested reader may consult [5,7,17,22] for a general survey for many variations of the lion-and-man problem. There also exists a substantial body of work on graph searching and pursuit-evasion in graphs [1,19,21], but our focus is on geometric environments.

The most relevant work to our research is the paper by Guibas et al. [11], which introduced a formal framework and analysis of visibility-based pursuit in complex polygonal environments. In order to make the problem tractable, however, Guibas et al. make one crucial simplifying assumption: *the evader loses if it is "seen" by any pursuer.* That is, the pursuers need to only detect the presence of the evader, and not physically catch it. With this weaker requirement of "capture," Guibas et al. manage to prove several interesting combinatorial bounds, including that $\Theta(\log n)$ pursuers in a simply-connected polygon, and $\Theta(h^{1/2} + \log n)$ pursuers in a polygon with h holes (obstacles), are always sufficient and sometimes necessary. Further work by Isler et al. [12] provided a randomized algorithm for the pursuers in which a single pursuer can locate the evader with high probability in a simply connected polygon, and $O(\sqrt{h})$ pursuers when there are h holes. Many other variations of the visibility-based pursuit-evasion have been studied over the years, including detection by a chain of guards [9] or guards with a limited field of view [10], but their focus remains detection, not capture.

Indeed, until recently, there had been little progress on extending these detection-of-evader bounds to physical capture of the evader. About a year ago, independently and simultaneously, two groups [3] and [14] proved that *if the location of the evader is always known to the pursuers*, e.g., using an ubiquitous camera network, then 3 pursuers are enough to win the game.[1] In a sense, this research suggests that "localization" of the evader is the more difficult part of the pursuit evasion, and the evader's power comes from its ability to "disappear" from the collective sights of all the pursuers.

The results of the current paper seek to compare the bounds on the number of pursuers required to locate or capture an evader. Indeed, we show that with only the *minimal assumption of equal speeds*, the bounds for capture are much worse, for example $\Theta(\sqrt{n})$ vs. $\Theta(\log n)$ pursuers for simply connected polygons. However, by further assuming the maximum movement speed of the players is bounded by the features of the environment, upper bounds matching those of Guibas et al. [11] are obtained. Finally, we show that the minimal condition of equal speeds is enough for a randomized capture algorithm to match the bounds for randomized localization of Isler et al. [12].

2. Capture in simple polygons

In this section, we establish the tight bound of $\Theta(n^{1/2})$ for the number of pursuers needed to capture the evader when the environment is a simply-connected (hole-free) polygon of n vertices. We begin with some definitions and preliminaries that are commonly used throughout the paper.

We use e and p_i, respectively, to denote the evader and the ith pursuer. We will often use this notation to denote their positions as well. The players' sensing model is *visibility-based*: two players see each other only when they are in line of sight. The pursuit occurs in discrete-time but continuous-space: the players move in alternate turns (with the evader making the first move), and are allowed to move anywhere within the polygon subject only to the speed constraint. On their

[1] The results of these two papers have since appeared in a joint journal article [4].