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Removing local extrema from imprecise terrains

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ABSTRACT

In this paper we consider imprecise terrains, that is, triangulated terrains with a vertical error interval in the vertices. In particular, we study the problem of *removing* as many local extrema (minima and maxima) as possible from the terrain; that is, finding an assignment of one height to each vertex, within its error interval, so that the resulting terrain has minimum number of local extrema. We show that removing only minima or only maxima can be done optimally in $O(n \log n)$ time, for a terrain with n vertices. Interestingly, however, the problem of finding a height assignment that minimizes the total number of local extrema (minima as well as maxima) is NP-hard, and is even hard to approximate within a factor of $O(\log \log n)$ unless $\mathrm{P} = \mathrm{NP}$. Moreover, we show that even a simplified version of the problem where we can have only three different types of intervals for the vertices is already NP-hard, a result we obtain by proving hardness of a special case of 2-Disjoint Connected Subgraphs, a problem that has lately received considerable attention from the graph-algorithms community.

1. Introduction

Digital terrain analysis is an important part of geographical information science, with applications in hydrology, geomorphology, visualization, and many other fields [7]. A popular structure for representing terrains is the *triangulated irregular network* (*TIN*), also known as *polyhedral terrain*. In this model, a terrain is represented by a planar triangulation with a height associated with each vertex. If we linearly interpolate the heights of the vertices, we also obtain a height at every other point in the plane, resulting in a bivariate, piecewise linear and continuous function, defining the surface of the terrain. A terrain in this model is also often called a 2.5-dimensional (or 2.5D) terrain.

1.1. Imprecision in terrains

In computational geometry it is usually assumed that the input data for any problem is correct and known exactly. In practice, this is unfortunately not the case. There are many sources of imprecision, the most prominent of which is the data acquisition itself. In terrain modeling, this is particularly relevant, because elevation data is collected by measuring devices that are ultimately error-prone. Often such devices produce heights with a known error bound or return a height interval rather than a fixed height value.

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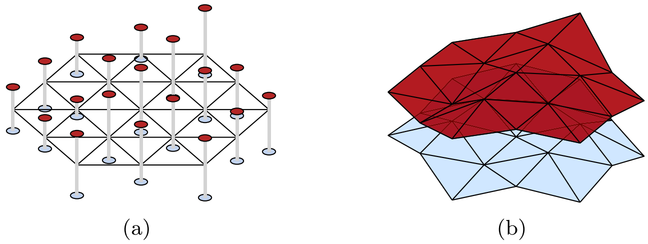

Fig. 1. (a) An example of an imprecise terrain. (b) The same terrain, shown by drawing the floor and the ceiling.

In order to handle the imprecision in terrains, we adopt the model used in [9,10,13], where the height of each terrain vertex is not precisely known, but only an interval of possible heights is available. This results in considerable freedom in the terrain, since the "real" terrain is unknown and any choice of a height for each vertex—as long as it is within its height interval—leads to a valid *realization* of the imprecise terrain. The large number of different realizations of an imprecise terrain leads naturally to the problem of finding one that is 'best' according to some criterion, or that avoids most or many instances of a certain type of unwanted feature (or artifact) from the terrain.

We note that, even though terrain data may contain error also in the x, y-coordinates, under this model we consider imprecision only in the z-coordinate. This simplifying assumption is justified by the fact that error in the x, y-coordinates will most likely produce elevation error. Moreover, often the data provided by commercial terrain data suppliers only reports the elevation error [6].

In the remainder of this paper, an *imprecise terrain* is a set of n vertical intervals in $\mathbb{R}^3$, together with a triangulation of the vertical projection. Fig. 1(a) shows an example. A *realization* of an imprecise terrain is a triangulated terrain that has the same triangulation in the projection, and exactly one vertex on each interval. An alternative way to view an imprecise terrain is by connecting the tops of all intervals into a terrain, which we call the *ceiling*, and the bottoms into a second terrain, which we call the *floor*. Then, a realization is a terrain that lives in the space left open between the floor and the ceiling. Fig. 1(b) shows this in the example.

1.2. Removing local extrema

A local minimum (or pit) is a location on a terrain that is surrounded by higher points. Similarly, a local maximum (or peak) is as a point surrounded by lower points. The term *local extrema* will be used to refer to both local minima and local maxima.

When terrains are used for studies of land erosion, landscape evolution, or hydrology, it is generally accepted that the majority of local extrema in the terrain model are spurious, caused by errors in the data or model production. A terrain model with many pits or peaks does not represent the terrain faithfully, and moreover, in the case of pits, it can create problems because water accumulates at them, affecting water flow routing simulations. For this reason the removal of local minima from terrain models is a standard preprocessing requirement for many uses of terrain models [26,23]. However, existing preprocessing routines make no attempt to relate the removed minima to knowledge about the imprecision in the terrain model, possibly causing major alterations to the data under study.

In this paper we attempt to solve the problems of removing as many local minima, maxima, or extrema as possible by moving the vertices of an *imprecise* terrain within their allowed height intervals. The rationale behind this is that if a pit (or peak) can be removed in this way, it is likely to be an artifact of the data, whereas if it cannot, it is more certain to be a 'real' pit (or peak). We define the *minimizing-minima*, the *minimizing-maxima*, and the *minimizing-extrema* problems on imprecise terrains, where we attempt to find a realization of an imprecise terrain (by placing the imprecise points within their intervals) that minimizes the number of *local minima*, *local maxima*, and *local extrema*, respectively.

It is important to note that a group of k connected vertices at the same height without any lower neighbor is considered to be only *one* local minimum. This is reasonable from the point of view of the application, and follows the definitions used in previous work [22]. In Section 4 we discuss what the implications of this modeling choice are for our results.

Regarding previous work, a lot of research has been devoted to the problem of removing local minima from (precise) terrains, especially in the geographic information science community, but also from more algorithmic points of view; we only provide a few relevant references here. Most of the literature assumes a raster (grid) terrain (e.g. [18,19,26]), and employs methods that are some type of "pit filling" technique, which consist in filling in depressions until they disappear (e.g. [4,18,26]). Some of the few exceptions are the methods in [3,19,21]. A few algorithms have been proposed for triangulated terrains, see for example the ones in [17,1]. The removal of local extrema has also been studied in the context of optimal higher order Delaunay triangulations [11,5]. In particular, Gudmundsson et al. [11] show that an optimal first-order Delaunay triangulation, with respect to the number of local minima and maxima, can be found in $O(n \log n)$ time. More related to this paper, Silveira and van Oostrum [22] study moving vertices vertically in order to remove all local minima with a minimum cost, but do not assume bounded intervals.