Computational Statistics and Data Analysis 83 (2015) 91–100

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Computation of optimum reliability acceptance sampling plans in presence of hybrid censoring

Ritwik Bhattacharya [a,*], Biswabrata Pradhan [a], Anup Dewanji [b]

[a] Statistical Quality Control and Operations Research Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata, PIN- 700108, India

[b] Applied Statistics Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata, PIN- 700108, India

ARTICLE INFO

Article history:
Received 6 May 2014
Received in revised form 15 September 2014
Accepted 1 October 2014
Available online 13 October 2014

Keywords:
Acceptance sampling
Consumer's risk
MIL-STD-105D
Monte-Carlo simulation
Optimum sampling plan
Producer's risk
Type-I hybrid censoring
Type-II hybrid censoring

ABSTRACT

The decision regarding acceptance or rejection of a lot of products may be considered through variables acceptance sampling plans based on suitable quality characteristics. A variables sampling plan to determine the acceptability of a lot of products based on the lifetime of the products is called reliability acceptance sampling plan (RASP). This work considers the determination of optimum RASP under cost constraint in the framework of hybrid censoring. Weibull lifetime models are considered for illustrations; however, the proposed methodology can be easily extended to any location-scale family of distributions. The proposed method is based on asymptotic results of the estimators of parameters of lifetime distribution. Hence, a Monte Carlo simulation study is conducted in order to show that the sampling plans meet the specified risks for finite sample size.

1. Introduction

Manufacturing houses or production industries often encounter the problem of selection of a lot or batch of products such as raw materials, components etc. that are outsourced from different vendors. The selection is important since sometimes the supplied materials do not match the specifications made by them. Acceptance sampling plan plays a key role in such situations for making decision whether to accept or reject a lot. A typical acceptance sampling plan is described as follows. The company receives a shipment of products from vendors. Some pre-specified quality characteristics are inspected through a sample from the shipment. Based on the information in the sample, the decision of accepting or rejecting the shipment is made. The major classification of acceptance sampling plans is either by attributes or by variables. In acceptance sampling by attributes, one need to specify a sample size n and an acceptability constant c, and the lot is considered to be acceptable if the number of defective items in the sample is less than or equal to c. On the other hand, in variables sampling plans, the distributional assumption on quality characteristics is required. Here, the quality characteristics are measured on some numerical scale and thus compared with a pre-specified acceptability constant in order to make a decision to accept or reject the lot. It is well-known that the variables sampling plans are more advantageous than the attributes sampling plans in the sense that the same operating characteristic curve can be obtained by using smaller sample size (See Montgomery, 2005). In this work, we consider reliability acceptance sampling plan (RASP) where quality characteristic is the lifetime of

* Corresponding author.
E-mail address: ritwik.bhatta@gmail.com (R. Bhattacharya).

http://dx.doi.org/10.1016/j.csda.2014.10.002

the product under consideration. It may be noted that reliability is a key feature of many products. The customer usually assesses the reliability of the product through a suitable life testing experiment after selecting a sample from the lot. Hence, a well planned RASP is important to the customer.

A life testing experiment, in general, is time consuming and expensive. In order to save time and money, different censoring schemes are applied to conduct a life testing experiment. A RASP under different censoring schemes like Type-I, Type-II, progressive censoring, etc., is discussed by various authors in the literature. For example, Schneider (1989), Lam (1994), Balasooriya et al. (2000), Balasooriya and Balakrishnan (2000), Balakrishnana et al. (2007), Balasooriya and Saw (1998), Huang and Wu (2008), Tsai et al. (2008), Fernández and Pérez-González (2012), Chen et al. (2004b), Wu and Huang (2012), Seidel (1997), Chen et al. (2004a), Rastogi and Tripathi (2013), etc. In the present work, we discuss the design of a RASP under hybrid censoring. Suppose that n items are put on a life testing experiment. The experiment is terminated after a prefixed r number of failures or at a prefixed time X_0 whichever is earlier. This is also known as Type-I hybrid censoring scheme (Type-I HCS). It is easy to see that Type-I and Type-II censoring schemes are special cases of Type-I HCS. When $r = n$, we get Type-I censoring scheme and when $X_0 = \infty$, it is Type-II censoring scheme. In this article, we first describe how to determine the sample size n and an acceptability constant, say l, for known r and X_0 to conduct a RASP under Type-I HCS satisfying the constraints specified by producer's and consumer's risks. The values of the design parameters r and X_0 are usually decided by experts. Therefore, the choice of (n, r, X_0) may not be optimal in a particular sense. We describe how to determine the optimum values of the design parameters (n, r, X_0) to conduct a RASP by minimizing a variance measure subject to a cost constraint in addition to meeting the specified producer's and consumer's risks. We provide an algorithm for computation of optimum RASP. Several optimum sampling plans are computed under various setups.

We describe, in Section 2, the development of sampling plans under Type-I HCS assuming Weibull lifetime distribution. We carry out a Monte Carlo simulation study to assess the finite sample properties of the maximum likelihood estimators and the accuracy of the Type-I hybrid censored sampling plans. We also consider the determination of sampling plans under Type-II hybrid censoring. The design of optimum sampling plan is discussed in Section 3. Finally, some concluding remarks are made in Section 4.

2. Hybrid censored reliability acceptance sampling plans

2.1. Hybrid censored data and maximum likelihood estimators

Suppose that the lifetime X of a testing unit follows the Weibull distribution with cumulative distribution function

$$F_X(x) = 1 - e^{-(\lambda x)^k}, \quad x > 0, \tag{1}$$

where $k > 0$ and $\lambda > 0$ are the shape and scale parameters, respectively. Considering the transformation $T = \ln X$, we have the extreme value distribution for T with the corresponding cumulative distribution function

$$F_T(t) = 1 - e^{-e^{\frac{t-\mu}{\sigma}}}, \quad -\infty < t < \infty, \tag{2}$$

where $-\infty < \mu < \infty$ and $\sigma > 0$ are the location and scale parameters given by $\mu = -\ln\lambda$ and $\sigma = 1/k$, respectively. Suppose that $X_1, X_2, \ldots, X_n$ be the lifetimes of n testing units which follow the Weibull distribution given by (1) and $T_1, T_2, \ldots, T_n$ be the corresponding log-lifetimes which follow the extreme value distribution given by (2). Let $T_{1:n} \le T_{2:n} \le \cdots \le T_{n:n}$ be the ordered failure times of those n units. In the framework of Type-I HCS, the number of failures and log-censoring time are denoted by D and $\tau = \min(T_{r:n}, T_0)$, respectively, where $T_0 = \ln X_0$. It is clear that both D and τ are random variables and the data is represented by $(T_{1:n}, T_{2:n}, \ldots, T_{D:n}, D)$. Note that, when $D = 0$, no failure is observed. For hybrid censored data, the likelihood function can be written as

$$L(\mu, \sigma) \propto \prod_{i=1}^{d} f_T(t_{i:n})(1 - F_T(\tau_0))^{n-d}, \tag{3}$$

where $t_{i:n}$, d and τ_0 denote the observed values of $T_{i:n}$, D and τ, respectively, and $f_T(\cdot)$ is the density function of T. The maximum likelihood estimates of μ and σ are obtained by maximizing (3). The Fisher information matrix for $\theta = (\mu, \sigma)$, using Park and Balakrishnan (2009), is given by

$$\mathcal{I}(\theta) = \int_{-\infty}^{T_0} \left\{\frac{\partial}{\partial\theta} \ln h_T(t)\right\}' \left\{\frac{\partial}{\partial\theta} \ln h_T(t)\right\} \sum_{i=1}^{r} f_{i:n}(t)\mathrm{d}t, \tag{4}$$

where $h_T(t)$ is the hazard function of T and $f_{i:n}(t)$ is the density function of $T_{i:n}$. Using (4), $\mathcal{I}(\theta)$ can be expressed as

$$\mathcal{I}(\theta) = \begin{pmatrix} \mathcal{I}_{11}(\theta) & \mathcal{I}_{12}(\theta) \\ \mathcal{I}_{21}(\theta) & \mathcal{I}_{22}(\theta) \end{pmatrix},$$