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Likelihood inference for generalized Pareto distribution

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ABSTRACT

A new methodological approach that enables the use of the maximum likelihood method in the Generalized Pareto Distribution is presented. Thus several models for the same data can be compared under Akaike and Bayesian information criteria. The view is based on a detailed theoretical study of the Generalized Pareto Distribution submodels with compact support.

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1. Introduction

The results produced by Balkema and de Haan (1974) and Pickands (1975) have widened the use of the Generalized Pareto Distribution (GPD) in extreme value theory as a model for tails; see McNeil et al. (2005), Beirlant et al. (2004), Finkenstädt and Rootzén (2003), Coles (2001) and Embrechts et al. (1997). However, several authors have highlighted the estimation problems that arise from this distribution; see Hosking and Wallis (1987), Castillo and Hadi (1997), Zhang and Stephens (2009). Currently, together with the maximum likelihood estimation (MLE), the methods of Zhang (2010) and Song and Song (2012) are the best alternatives. The main issue with MLE is that for some datasets the likelihood function appears to have no local maximum.

The motivation for this study lies in the fact that, despite its weaknesses, we need the MLE in many inference procedures. The adjustments of various models to the same data are commonly compared with the Akaike (AIC) and Bayesian (BIC) information criteria and with the likelihood ratio test (LRT) in nested models, all of which are based on the MLE; see Clauset et al. (2009). It is also common to use the MLE in goodness of fit tests as in Choulakian and Stephens (2001). A summary of the theoretical results for MLE in the GPD can be found in Davison and Smith (1990). After that, many authors report that the MLE exists for large samples provided $\kappa < 1$ and is asymptotically efficient provided $\kappa < 0.5$. For $\kappa > 1$ with probability tending to 1 there is no local maximum. This article goes a step further; see Theorem 1 and Proposition 4.

In Section 2, we propose a fast, simple and stable algorithm, which always provides an estimate of the MLE when possible. This algorithm is compared with those of Zhang (ZSE) and Song and Song (SSE), in terms of efficiency and bias, where neither of existing algorithms has been shown to dominate. According to the behavior of the probability density functions of the GPD, we can distinguish three submodels that are separated by the exponential distribution ($\kappa = 0$) and the uniform distribution ($\kappa = 1$). The study highlights the problem of misspecification of the submodel. Running a simulation from one of the three

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Table 1
Percentages of the classification for each submodel for some sample size and values of κ. The model A corresponds to the submodel of GPD for $\kappa \le 0$, the model B for $0 \le \kappa \le 1$ and the model C for $\kappa \ge 1$. The bold values correspond to well-classified categories.

	A			B						C		
κ	−0.1			0.1			0.9			1.1		
n	A	B	C	A	B	C	A	B	C	A	B	C
ZSE												
15	**65.8**	34.1	0.1	44.1	**55.5**	0.4	0.9	**76.5**	22.6	0.4	57.7	**41.9**
25	**71.0**	29.0	0.0	41.7	**58.3**	0.0	0.1	**79.8**	20.1	0.0	52.7	**47.3**
50	**76.5**	23.6	0.0	30.9	**69.1**	0.0	0.0	**83.7**	16.3	0.0	40.5	**59.5**
100	**84.2**	15.8	0.0	19.9	**80.1**	0.0	0.0	**90.7**	9.3	0.0	31.0	**69.0**
SSE												
15	**85.3**	14.5	0.3	73.8	**25.6**	0.7	20.7	**50.0**	29.4	14.7	40.7	**44.6**
25	**73.4**	26.6	0.0	54.2	**45.8**	0.0	3.7	**61.0**	35.3	1.8	40.4	**57.8**
50	**67.5**	32.6	0.0	39.4	**60.6**	0.0	0.1	**69.3**	30.6	0.0	34.4	**65.6**
100	**70.1**	**30.0**	0.0	29.1	**70.9**	0.0	0.0	**77.1**	22.9	0.0	27.3	**72.7**
MLE												
15	**41.4**	50.2	8.4	18.8	**64.6**	16.6	0.1	**11.2**	88.8	0.0	4.1	**95.9**
25	**49.2**	49.9	0.9	17.4	**80.1**	2.5	0.0	**16.1**	83.9	0.0	4.8	**95.2**
50	**63.0**	37.0	0.0	15.2	**84.8**	0.0	0.0	**30.1**	69.9	0.0	4.8	**95.2**
100	**75.0**	25.0	0.0	9.7	**90.3**	0.0	0.0	**49.8**	50.2	0.0	4.4	**95.6**

GPD submodels, the estimated parameters can easily belong to another, as shown in Table 1. The best correct classification results are always obtained with MLE or ZSE.

Section 3 gives some mathematical results that provide precise arguments to explain the anomalous behavior of the likelihood function when sampling from the GPD distribution for positive values of κ. Theorem 2 proves that there is a global MLE for the submodel $0 \le \kappa \le 1$. Propositions 2 and 3 show that the likelihood function extends continuously up to the boundary and reaches the likelihood of the exponential and uniform distributions. Hence, the AIC and BIC criteria can be used in this case. However, it is proved that for $\kappa > 1$ the likelihood function is unbounded and reaches infinity, in a half-line of the parameter space. Hence, mathematically, neither AIC nor BIC can be used properly for $\kappa > 1$.

In Section 4, the numerical and analytical results of the previous sections lead us to suggest a new methodological approach that emphasizes the role of transition points $\kappa = 0$ and $\kappa = 1$. Two of the most commented examples of extreme values theory are discussed in Section 5. Finally, Section 6 shows a summary of the main conclusions.

2. Computing the maximum likelihood estimation

The Generalized Pareto Distribution (GPD), named by Pickands (1975), is a two-parameter family of distributions, with the cumulative distribution function given by

$$F(x; \kappa, \psi) = 1 - (1 - \kappa x/\psi)^{1/\kappa}, \tag{1}$$

where $\psi > 0$ and κ are the scale and shape parameters, respectively. For $\kappa > 0$ the range of x is $0 < x < \psi/\kappa$ and for $\kappa < 0$ the range is $x > 0$. Let us define σ as ψ/κ. The GPD becomes the uniform distribution for $\kappa = 1$ and the exponential distribution for $\kappa = 0$ (taken as the limit). The GPD has mean $\psi/(1 + \kappa)$ and variance $\psi^2/[(1 + \kappa)^2(1 + 2\kappa)]$ provided $\kappa > -0.5$. Note that the coefficient of variation is independent of the scale parameter, since it is given by

$$cv = 1/\sqrt{(1 + 2\kappa)}. \tag{2}$$

The probability density function of the GPD is given by

$$f(x; \kappa, \psi) = \psi^{-1}(1 - \kappa x/\psi)^{1/\kappa - 1}. \tag{3}$$

Fig. 1 shows the importance of the uniform distribution ($\kappa = 1$) in the GPD, which is between the monotone decreasing probability density functions ($\kappa < 1$) and the monotone increasing probability density functions tending to infinity at the upper support ($\kappa > 1$).

Let $x_1, \ldots, x_n$ be a random sample from the GPD and let $x_{(1)} \le \cdots \le x_{(n)}$ be the order statistics of the sample. The log-likelihood is given by

$$l(\kappa, \psi) = n\left(-\log(\psi) + (1/\kappa - 1)\frac{1}{n}\sum_{i=1}^{n}\log(1 - \kappa x_i/\psi)\right), \tag{4}$$

where $\psi > 0$ for $\kappa \le 0$ and $\psi > \kappa x_{(n)}$ for $\kappa > 0$.

The log-likelihood function parameterized by κ and σ is denoted by l^*, that is $l^*(\kappa, \sigma) = l(\kappa, \kappa\sigma)$, where $\sigma < 0$ for $\kappa \le 0$ and $\sigma > x_{(n)}$ for $\kappa > 0$. Note that σ is the right endpoint for the distributions with bounded support. This parameterizing

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