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Simultaneous confidence intervals for ratios of means of several lognormal distributions: A parametric bootstrap approach

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HIGHLIGHTS

- We present parametric bootstrap (PB) simultaneous confidence intervals.
- Our PB procedure is different from a previous PB procedure which performs poorly.
- Using simulation, we compare the PB procedure with three other procedures.
- The coverage probability of the PB method is close to the nominal confidence level.
- The PB procedure consistently outperforms other procedures.

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ABSTRACT

For constructing simultaneous confidence intervals for the ratios of means of several lognormal distributions, we propose a new parametric bootstrap method, which is different from an inaccurate parametric bootstrap method previously considered in the literature. Our proposed method is conceptually simpler than other proposed methods, which are based on the concepts of generalized pivotal quantities and fiducial generalized pivotal quantities. Also, our extensive simulation results indicate that our proposed method consistently performs better than other methods: its coverage probability is close to the nominal confidence level and the resulting intervals are typically shorter than the intervals produced by other methods.

1. Introduction

The lognormal distribution is widely used for analyzing positive right-skewed data; see, e.g., Oldham (1953), Koch (1966), Zhou et al. (1997), Lacey et al. (1997), Julius and Debarnot (2000), and Shen et al. (2006). We use the notation $X \sim LN(\mu, \sigma^2)$ to denote the lognormal distribution with parameters μ and σ^2; i.e. $Y = \ln(X) \sim N(\mu, \sigma^2)$. The main parameter of interest is usually the mean:

$$E(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right).$$

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Suppose that random samples are taken from k lognormal distributions, $LN\left(\mu_i, \sigma_i^2\right)$, $i = 1, \ldots, k$. Let $m_i = \exp\left(\mu_i + \sigma_i^2/2\right)$ denote the mean of the i-th sample, and let

$$\eta_i = \ln(m_i) = \mu_i + \sigma_i^2/2.$$

We consider the problem of constructing simultaneous confidence intervals (SCIs) for the ratios of the means m_i/m_j $(i \neq j)$ or, equivalently, for $\eta_i - \eta_j$, $i, j = 1, \ldots, k$ $(i \neq j)$, when the σ_i^2 are possibly unequal. This problem arises in the context of multiple comparisons of several treatments when the response variable is positive and has a right-skewed distribution, as is often the case in biological or medical studies; for example in bioequivalence studies for comparing several drug formulations; see Hannig et al. (2006) and Schaarschmidt (2013).

Inferences about the mean of a lognormal distribution, based on a single random sample, have been considered by Land (1971, 1972, 1973, 1975, 1988), Angus (1994), Zhou et al. (1997), and Krishnamoorthy and Mathew (2003). The problem of comparing two lognormal means ($k = 2$) was considered by Zhou et al. (1997), Zhou and Tu (2000), Krishnamoorthy and Mathew (2003), and Chen and Zhou (2006). Li (2009) proposed a procedure for testing the equality of several lognormal means. For constructing SCI for the ratios of means, Hannig et al. (2006) proposed a procedure based on fiducial generalized pivotal quantities (FGPQ) and Schaarschmidt (2013) proposed two procedures based on generalized pivotal quantities (GPQ).

In this paper, we propose a parametric bootstrap (PB) approach for constructing SCI for the ratios of means. The PB approach has been successfully used in other contexts (mainly testing problems) involving nuisance parameters; see, e.g., Krishnamoorthy et al. (2007), Ma and Tian (2009), Tian et al. (2009), Krishnamoorthy and Lu (2010), Li et al. (2011), Xu et al. (2013), and Sadooghi-Alvandi and Jafari (2013). In fact Zhou and Tu (2000) and Chen and Zhou (2006) considered a PB procedure for constructing confidence intervals for the ratio of the means of two lognormal distributions ($k = 2$), but their proposed confidence intervals perform rather poorly. In view of this, the PB approach has not been considered for the k-sample problem: as Schaarschmidt (2013, p. 269) commented regarding the PB approach: "The simulation results of Chen and Zhou (2006) do not motivate to consider this approach any further". Our proposed PB procedure, however, is different from the PB procedure considered by Chen and Zhou (2006) and is more accurate. In fact, our extensive simulation results indicate that it performs better than other methods: its coverage probability is close to the nominal confidence level and the resulting intervals are typically shorter than the intervals produced by other methods.

Our proposed PB procedure is presented in Section 2 and our simulation results are presented in Section 3. In our simulations, we first compare our PB procedure with the PB procedure considered by Zhou and Tu (2000) and Chen and Zhou (2006) for the case $k = 2$. We then consider the case $k \geq 3$ and compare our proposed PB procedure with the procedures proposed by Hannig et al. (2006) and Schaarschmidt (2013). Our simulation results indicate that our proposed PB procedure consistently performs better than these procedures.

2. Parametric bootstrap simultaneous confidence intervals

Let $X_{i1}, \ldots, X_{in_i}$ denote a random sample of size n_i from the lognormal distribution $LN\left(\mu_i, \sigma_i^2\right)$, $i = 1, \ldots, k$ and let

$$\eta_i = \mu_i + \frac{\sigma_i^2}{2}. \tag{2.1}$$

We shall be interested in constructing SCI for $(\eta_i - \eta_j)$, $i, j = 1, \ldots, k$ $(i \neq j)$. Let

$$Y_{ij} = \ln(X_{ij}), \qquad \bar{Y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}, \qquad S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2, \quad i = 1, \ldots, k.$$

It is well known that $\bar{Y}_i$ and S_i^2 are independent and

$$\bar{Y}_i \sim N\left(\mu_i, \frac{\sigma_i^2}{n_i}\right), \qquad \frac{(n_i - 1)S_i^2}{\sigma_i^2} \sim \chi^2_{(n_i-1)}, \quad i = 1, \ldots, k \tag{2.2}$$

(where χ_r^2 denotes a chi-square distribution with r degrees of freedom). The natural (unbiased) estimator of η_i is

$$\hat{\eta}_i = \bar{Y}_i + \frac{S_i^2}{2}, \quad i = 1, \ldots, k.$$

Note that

$$\mathrm{Var}(\hat{\eta}_i - \hat{\eta}_j) = \frac{\sigma_i^2}{n_i} + \frac{\sigma_i^4}{2(n_i - 1)} + \frac{\sigma_j^2}{n_j} + \frac{\sigma_j^4}{2(n_j - 1)}, \quad i \neq j \tag{2.3}$$

and an unbiased estimator of (2.3) is

$$V_{ij} = \frac{S_i^2}{n_i} + \frac{S_i^4}{2(n_i + 1)} + \frac{S_j^2}{n_j} + \frac{S_j^4}{2(n_j + 1)};$$