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EM algorithm for one-shot device testing under the exponential distribution

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ABSTRACT

The EM algorithm is a powerful technique for determining the maximum likelihood estimates (MLEs) in the presence of binary data since the maximum likelihood estimators of the parameters cannot be expressed in a closed-form. In this paper, we consider one-shot devices that can be used only once and are destroyed after use, and so the actual observation is on the conditions rather than on the real lifetimes of the devices under test. Here, we develop the EM algorithm for such data under the exponential distribution for the lifetimes. Due to the advances in manufacturing design and technology, products have become highly reliable with long lifetimes. For this reason, accelerated life tests are performed to collect useful information on the parameters of the lifetime distribution. For such a test, the Bayesian approach with normal prior was proposed recently by Fan et al. (2009). Here, through a simulation study, we show that the EM algorithm and the mentioned Bayesian approach are both useful techniques for analyzing such binary data arising from one-shot device testing and then make a comparative study of their performance and show that, while the Bayesian approach is good for highly reliable products, the EM algorithm method is good for moderate and low reliability situations.

1. Introduction

In one-shot device testing, since one can observe only the conditions of the devices at several specified times, the binary data are not only left censored but are also right censored. Such data are also called *current status data*. For instance, Fan et al. (2009) considered the reliability of electro-explosive devices wherein the devices are detonated by inducing a current to excite inner powder. Note that those devices cannot be used any further after detonation. No matter whether the detonation is successful or not, the lifetime of the device cannot be recorded. Also, due to high reliability of such devices, there will be very few failures or even no failure within a limited time, under normal conditions. This would pose a problem in the analysis if only few or no failures are observed. For this reason, accelerated life tests are often used by adjusting a controllable factor such as temperature in order to have more devices fail in the experiment. Moreover, accelerated life testing would shorten the experimental time and also help reduce the experimental cost. Meeker et al. (1998) studied accelerated degradation data on integrated-circuit (IC) devices and used the accelerated degradation model to develop inference and prediction about its lifetime distribution at standard operating temperature. In their article, they have mentioned many applications of accelerated life tests and also described methods to analyze such kinds of data. Lu et al. (1996) compared degradation analysis and traditional failure time analysis in terms of asymptotic variance of estimators of a quantile of the lifetime distribution. Rodrigues et al. (1993) presented two approaches based on the likelihood ratio statistics and the posterior Bayes factor for comparing several exponential accelerated life models.

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Table 1
The number of failures recorded under temperatures 35, 45, 55 (in degrees) at inspection times 10, 20, 30, respectively, in a one-shot device testing.

	$T_1 = 10$	$T_2 = 20$	$T_3 = 30$
$w_1 = 35$	$n_{11} = 3$	$n_{12} = 3$	$n_{13} = 7$
$w_2 = 45$	$n_{21} = 1$	$n_{22} = 5$	$n_{23} = 7$
$w_3 = 55$	$n_{31} = 6$	$n_{32} = 7$	$n_{33} = 9$

Fan et al. (2009) developed the Bayesian approach for one-shot device testing along with an accelerating factor, in which the failure time of the devices is assumed to follow an exponential distribution. In spite of small sample sizes in their simulation study, their Bayesian estimator incorporating normal prior yields accurate inference on the parameters, the reliability as well as the mean lifetime under normal temperature. In their development, a strong assumption that the prior information on the success rate is very reliable was made, which means that the reliability estimates attained are based on the information that it is close to the true value. In their simulation study, the prior information was generated from a distribution which is around the true value and with a small error.

In this article, the EM algorithm is developed for finding the maximum likelihood estimates (MLEs) of the model parameters and this method is then compared with the above mentioned Bayesian approach. It is shown that the proposed method yields quite reliable and efficient estimates. Much work has been done on the estimation of parameters in incomplete data problems through the EM algorithm. For instance, Ng et al. (2002) developed inference for the lognomal and Weibull distributions based on progressively censored data. Nandi and Dewan (2010) considered the bivariate Weibull distribution under random censoring using the EM algorithm and analyzed soccer data from the UEFA Champions League. Chen et al. (2009) also presented the EM algorithm based on a proportional hazards frailty model for analyzing a bivariate current status data arising from a tumorigenicity experiment. In this article, we will evaluate the developed EM algorithm for the problem considered and then compare its performance with the Bayesian approach with an attainable prior information, and demonstrate that the EM algorithm is quite useful for analyzing such data.

The rest of this article is organized as follows. Section 2 first presents the form of the data and the model with an illustrative data. Section 3 develops the EM algorithm for the analysis of data. Section 4 describes briefly the Bayesian approach of Fan et al. (2009) and in Section 5, a simulation study is conducted to compare the performance of the two methods in estimating the model parameters, the reliability at different times, and the mean failure time under normal temperature. In Section 6, we present a numerical example to illustrate the inferential results developed here. Finally, some concluding remarks are made in Section 7.

2. Model description

Consider a reliability testing experiment in which $J \times K$ devices are placed under temperature W_i, of which K devices are tested at time T_j, where $i = 1, 2, \dots, I$ and $j = 1, 2, \dots, J$. It is worth noting that a successful detonation occurs if its lifetime is beyond the inspection time, whereas the lifetime will be before the inspection time if the detonation is a failure. For each temperature W_i, the number of failures n_{ij} is then recorded at each inspection time T_j. For the example illustrated in Table 1, 30 devices were tested at temperatures $W_i = 35, 45, 55$ each, of which 10 units were detonated at times $T_j = 10, 20, 30$ each. The number of failures observed is summarized in a 3×3 table in Table 1. There were in all 48 failures out of a total of 90 devices that were tested in this one-shot device testing experiment.

Johnson et al. (1994, 1995) have presented booklength accounts on many lifetime distributions that are useful for reliability analysis. We assume here that the true lifetimes t_{ijk}, where $i = 1, 2, \dots, I, j = 1, 2, \dots, J, k = 1, 2, \dots, K$, are independent and identically distributed exponential random variables with probability density function (pdf)

$$f(t) = \lambda e^{-\lambda t},$$

where λ is the failure rate. Here, we relate the parameter λ to an accelerating factor of temperature W_i through a log-linear link function as follows:

$$\lambda_{W_i} = \alpha_0 e^{\alpha_1 W_i}, \quad \alpha_0, W_i > 0.$$

Let p_{ij} denote the associated survival probability under temperature W_i at time T_j. We then have

$$p_{ij} = 1 - F(T_j|\lambda_{W_i}) = \exp(-\lambda_{W_i} T_j) = \exp(-\alpha_0 e^{\alpha_1 W_i} T_j).$$

Now, given the data on K, $\mathbf{n} = \{n_{ij},\ i = 1, 2, \dots, I,\ j = 1, 2, \dots, J\}$ collected at the temperatures $\mathbf{W} = \{W_i,\ i = 1, 2, \dots, I\}$ and the inspection times $\mathbf{T} = \{T_j,\ j = 1, 2, \dots, J\}$, the likelihood function of α_0 and α_1 is given by

$$\begin{aligned} L(\alpha_0, \alpha_1 \mid K, \mathbf{n}, \mathbf{T}, \mathbf{W}) &= \prod_{i=1}^{I} \prod_{j=1}^{J} p_{ij}^{K-n_{ij}} (1 - p_{ij})^{n_{ij}} \\ &= \prod_{i=1}^{I} \prod_{j=1}^{J} \exp(-\alpha_0 (K - n_{ij}) e^{\alpha_1 W_i} T_j) \{1 - \exp(-\alpha_0 e^{\alpha_1 W_i} T_j)\}^{n_{ij}}. \end{aligned}$$