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Parameter estimations for generalized exponential distribution under progressive type-I interval censoring

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ABSTRACT

The estimates, via maximum likelihood, moment method and probability plot, of the parameters in the generalized exponential distribution under progressive type-I interval censoring are studied. A simulation is conducted to compare these estimates in terms of mean squared errors and biases. Finally, these estimate methods are applied to a real data set based on patients with plasma cell myeloma in order to demonstrate the applicabilities.

1. Introduction

The two-parameter generalized exponential (GE) distribution has a probability density function, a distribution function and a hazard function as follows:

$$f(t,\theta) = \alpha\lambda(1-e^{-\lambda t})^{\alpha-1}e^{-\lambda t}, \tag{1.1}$$

$$F(t,\theta) = (1-e^{-\lambda t})^{\alpha} \tag{1.2}$$

and

$$h(t,\theta) = \frac{\alpha\lambda(1-e^{-\lambda t})^{\alpha-1}e^{-\lambda t}}{1-(1-e^{-\lambda t})^{\alpha}} \tag{1.3}$$

where $\theta = (\alpha,\lambda)$, $\alpha > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter. When $\alpha = 1$, the GE distribution defined above reduces to the conventional exponential distribution. If $\alpha < 1$, the density function (1.1) is decreasing and if $\alpha > 1$, the density function (1.1) is a unimodal function. Similarly to a Weibull distribution, the hazard function of a GE distribution could be increasing, decreasing or constant depending on the shape parameter α. The GE distribution was introduced by Mudholkar and Srivastava (1993) as an alternative to the commonly used gamma and Weibull distributions. Since then, GE distribution has been studied by many authors, for example, Gupta and Kundu (1999, 2001a,b, 2002), Raqab and Ahsanullah (2001), Jaheen (2004), Raqab and Madi (2005), Sarhan (2007) and Zheng (2002). Gupta and Kundu (2001a, 2003) mentioned that the two-parameter GE distribution could be used quite effectively in analyzing many lifetime data, particularly, in place of the two-parameter gamma or two-parameter Weibull distribution and in many situations the two-parameter GE

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distribution could provide a better fit than the two-parameter Weibull distribution. Gupta and Kundu (2001b) also studied many different methods of parameter estimations which included maximum likelihood estimates, estimates of moment method and estimates by the method of probability plot based on a complete random sample. An extensive survey of some recent developments for the two-parameter GE distribution based on a complete random sample can be found from Gupta and Kundu (2007).

In industrial life testing and medical survival analysis, it is very often that object is lost or withdrawn before failure or the object lifetime is only known within an interval. Hence, the obtained sample is called a censored sample (or an incomplete sample). The most common censoring schemes are type-I censoring, type-II censoring and progressive censoring. The life testing is ended at a pre-scheduled time for the type-I censoring and for the type-II, the life testing is ended whenever the number of lifetimes is reached. In the type-I and the type-II censoring schemes, the tested items are allowed to be withdrawn only at the end of life testing. In the progressive censoring scheme, the tested items are allowed to be withdrawn at some other times before the end of life testing. See Balakrishnan and Aggarwala (2000) for more information about progressive censoring combining with type-I or type-II and the applications. Based on an incomplete sample from the GE distribution, Sarhan (2007) developed an inference process for a competing risk model, and Pradhan and Kundu (2008) obtained a statistical inference for a progressively censored sample. Their results are for the type-II censoring scheme. Therefore, the results from Sarhan (2007) as well as the results from Pradhan and Kundu (2008) are for the incomplete samples which must contain a fixed number of lifetimes.

Aggarwala (2001) introduced type-I interval and progressive censoring and developed the statistical inference for the exponential distribution based on progressively type-I interval censored data. Under progressive type-I interval censoring, observations are only known within two consecutively pre-scheduled times and items would be allowed to withdraw at pre-scheduled time points. Ng and Wang (2009) introduced the concept of progressive type-I interval censoring to the Weibull distribution and compared many different estimation methods for two parameters in the Weibull distribution via simulation. In this paper, we follow a pattern very similar for the Weibull distribution to study maximum likelihood estimates, estimates via moment methods and estimates via probability plot for the two parameters in the GE distribution under the progressive type-I interval censoring.

The rest of this article is organized as follows. Section 2 introduces the progressive type-I interval censoring into the GE distribution followed by the theoretical backgrounds and methods for its parameter estimations. In Section 3, a simulation study is conducted to compare the performances of these estimation methods based on the mean squared error (MSE) and bias. In Section 4, the application to a real data set is discussed and conclusions are given in Section 5.

2. Data, likelihood and parameter estimations

2.1. Progressively type-I interval censored data

Let n items be placed on a life testing simultaneously at time $t_0 = 0$ and under inspection at m pre-specified times $t_1 < t_2 < \cdots < t_m$ where t_m is the scheduled time to terminate the experiment. At the ith inspection time, t_i, the number, X_i, of failures within $(t_{i-1}, t_i]$ is recorded and R_i surviving items are randomly removed from the life testing, for $i = 1, 2, \ldots, m$. Since the number, Y_i, of surviving items is a random variable and the exact number of items withdrawn should not be greater than Y_i at time schedule t_i, R_i could be determined by the pre-specified percentage of the remaining surviving units at t_i for given $i = 1, 2, \ldots, m$. For example, given pre-specified percentage values, $p_1, \ldots, p_{m-1}$ and $p_m = 1$, for withdrawing at $t_1 < t_2 < \cdots < t_m$, respectively, $R_i = \lfloor p_i y_i \rfloor$ at each inspection time t_i where $i = 1, 2, \ldots, m$. Therefore, a progressively type-I interval censored sample can be denoted as $\{X_i, R_i, t_i\}$, $i = 1, 2, \ldots, m$, where sample size $n = \sum_{i=1}^{m}(X_i + R_i)$. If $R_i = 0, i = 1, 2, \ldots, m-1$, then the progressively type-I interval censored sample is a type-I interval censored sample, $X_1, X_2, \ldots, X_m, X_{m+1} = R_m$.

2.2. Likelihood function

Given a progressively type-I interval censored sample, $\{X_i, R_i, t_i\}$, $i = 1, 2, \ldots, m$, of size n, from a continuous lifetime distribution with distribution function, $F(T, \theta)$, where θ is the parameter vector, the likelihood function can be constructed as follows (see for example Aggarwala, 2001):

$$\begin{aligned} L(\theta) &\propto [F(t_1,\theta)]^{X_1}[1-F(t_1,\theta)]^{R_1} \times [F(t_2,\theta)-F(t_1,\theta)]^{X_2}[1-F(t_2,\theta)]^{R_2} \\ &\quad \times \cdots \times [F(t_m,\theta)-F(t_{m-1},\theta)]^{X_m}[1-F(t_m,\theta)]^{R_m} \\ &= \prod_{i=1}^{m}[F(t_i,\theta)-F(t_{i-1},\theta)]^{X_i}[1-F(t_i)]^{R_i} \end{aligned} \quad (2.1)$$

where $t_0 = 0$. It can be seen easily that if $R_1 = R_2 = \cdots = R_{m-1} = 0$, the likelihood function (2.1) reduces to the corresponding likelihood function for the conventional type-I interval censoring. The maximum likelihood estimate (MLE) for the parameter can be carried out by maximizing this likelihood function in (2.1). Generally, there is hardly a closed form for the MLE and therefore an iterative numerical search could be used to obtain the MLE from the above likelihood function.