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Bounding the locus of the center of mass for a part with shape variation

Fatemeh Panahi [1], A. Frank van der Stappen *

Department of Information and Computing Sciences, Utrecht University, PO Box 80089, 3508 TB Utrecht, The Netherlands

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ABSTRACT

The shape and center of mass of a part are crucial parameters to algorithms for planning automated manufacturing tasks. As industrial parts are generally manufactured to tolerances, the shape is subject to variations, which, in turn, also cause variations in the location of the center of mass. Planning algorithms should take into account both types of variation to prevent failure when the resulting plans are applied to manufactured incarnations of a model part.

We study the relation between variation in part shape and variation in the location of the center of mass for a part with uniform mass distribution. We consider a general model for shape variation that only assumes that every valid instance contains a shape P_I while it is contained in another shape P_E. We characterize the worst-case displacement of the center of mass in a given direction in terms of P_I and P_E. The characterization allows us to determine an adequate polytopic approximation of the locus of the center of mass. We also show that the worst-case displacement is small if P_I is convex and fat and the distance between the boundary of P_E and P_I is bounded.

1. Introduction

Many automated part manufacturing tasks involve manipulators that perform physical actions—such as pushing, squeezing [1], or pulling [2]—on the parts. Over the past two decades, researchers in robotics in general and algorithmic automation in particular have thoroughly studied the effect of physical actions as well as their potential role in accomplishing high-level tasks like orienting or sorting. It is evident that shape and—in many cases (see e.g. [1,3–7])—location of the center of mass are important parameters in determining the effect of a physical action on a part.

Industrial parts are always manufactured to tolerances as no production process is capable of delivering parts that are perfectly identical. Tolerance models [8,9] are therefore used to specify the admitted variations with respect to the CAD model. A consequence of these variations [10,11] is that actions that are computed on the basis of a CAD model of a part may easily lead to different behavior when executed on a manufactured incarnation of that part, and thus to failure to accomplish the higher-level task. It is important to note that the shape variations not only directly affect the behavior of the part but indirectly as well because they also cause a displacement of the center of mass of the part.

To extend the planning algorithms to imperfect manufactured incarnations, it is important to understand the effects of variations and take them into account during planning. Larger variations in part shape and center-of-mass location inevitably

* Corresponding author.
E-mail addresses: F.Panahi@uu.nl (F. Panahi), A.F.vanderStappen@uu.nl (A.F. van der Stappen).

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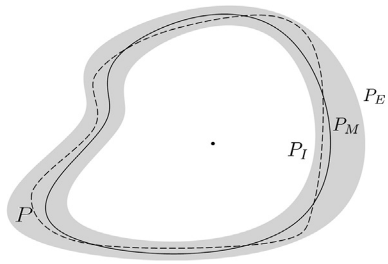

Fig. 1. A family of shapes specified by a subshape P_I and a supershape P_E of a model part P_M, along with a valid instance $P \in S(P_I, P_E)$.

result in a larger range of possible part behaviors, which reduces the likeliness that a manufacturing task can be accomplished. Therefore we will study how variations in part shape influence the location of the center of mass. (Note that variations in shape and center of mass are not the only sources of uncertainty in robotics. Additional uncertainty can result from the inaccuracy of the actuators and manipulators [12] and sensors [13].)

Several geometric approaches have been proposed to overcome the problems occurring in the presence of uncertainty and to smooth the effects of errors. Among the existing approaches are the model of ϵ-geometry [14], tolerance and interval geometry [15,16] and region-based models [17]. Generally, in all these models an uncertain point is represented by a region in which it may vary. The model of ϵ-geometry assumes that a point can vary within a disk of radius ϵ. Tolerance and interval-geometry take into account coordinate errors which results in an axis-aligned rectangular region in which a point can vary. In general, region-based models represent a point by any convex region. After modeling uncertainty as a point surrounded by a region, it is possible to study worst (and best) cases for a problem under the specific uncertainty model.

As observed before, variation of the shape causes variation of the center of mass of a part. The locus of the centroid of a set of points with approximate weights has been studied by Bern et al. [18]. Akella et al. [19] estimated the locus for a polygon under the ϵ-geometry model [19]. The problem of finding the locus of the center of mass of a part with shape variation and uniformly distributed mass has been mentioned as an open problem [11,19]. Akella et al. [19] studied rotating a convex polygon whose vertices and center of mass lie inside predefined circles centered at their nominal locations. The problem of orienting a part by fences has been studied by Chen et al. [11]. They define disk and square regions for the vertices of a part and proposed a method for computing the maximum allowable uncertainty radius for each vertex. They also discussed in a more general way the key role of the center of mass and the successfulness of part feeding (or orienting) algorithms in a setting of shape variation. Chen et al. [20] presented algorithms for squeezing and pushing problems. Kehoe et al. [21] explored cloud computing in a context of grasping and push-grasping under shape variation.

All the previous models for shape variation only allow the vertices to vary. In this paper we use a more general model for shape variation. For given shapes P_I and P_E such that $P_I \subseteq P_E$ we consider the family of shapes P satisfying $P_I \subseteq P \subseteq P_E$. In the practical setting of toleranced parts the shapes P_I and P_E will be fairly similar. We will show in Section 3 that the valid instance that yields the largest displacement of the center of mass in a given direction is a shape that combines a part of P_I with a part of P_E. The corresponding displacement is computable in $O(n \log n)$ time where n is the complexity of P_I and P_E; it can be used to obtain a k-facet outer approximation of the set of all possible loci of the center of mass in $O(kn \log n)$ steps.

In Section 4, we will study the size of the set of possible center-of-mass loci. Fatness of the objects under consideration has led to lower combinatorial complexities and more efficient algorithms for various problems, including union complexities [22], motion planning [23], hidden surface removal [24], and range searching [25]. Here we show that fatness and convexity of P_I together with the assumption that no point in P_E has a distance larger than ϵ to some point in P_I leads to a bound on the distance between the centers of mass of any two valid instances of a part which is proportional to ϵ and the fatness of P_I.

2. Preliminaries

In this section, we first present a general model for shape variations, then review the notion of a center of mass, and finally introduce a few notions that allow us to characterize the shapes that maximize the displacement of the center of mass. Let $P_M \subset \mathbb{R}^d$ be the model part, with $d = 2$ or $d = 3$. The part P_M has a uniform mass distribution.

No production process ever delivers parts that are perfectly identical to the model part P_M and therefore industrial parts are manufactured to tolerances. We use a very general model for permitted shape variations that only requires that any manufactured instance of P_M contains a given subshape P_I of P_M while it is contained in a supershape P_E of P_M. As a result, the set of acceptable instances of P_M is a family of shapes $S(P_I, P_E) = \{P \subset \mathbb{R}^d \mid P_I \subseteq P \subseteq P_E\}$ for given P_I and P_E satisfying $P_I \subseteq P_M \subseteq P_E$. In other words, the boundary ∂P of an instance $P \in S(P_I, P_E)$ should be entirely contained in $Q = P_E - int(P_I)$ where $int(P)$ denotes the interior of the set P. The region Q is referred to as the *tolerance zone*. The objects P_I and P_E are assumed to be closed semi-algebraic sets with a total of n boundary features. (Fig. 1 shows an example of a model part P_M, shapes P_I and P_E, and a valid instance $P \in S(P_I, P_E)$.) We denote by $COM(P_I, P_E)$ the set of all centers of mass of instances $P \in S(P_I, P_E)$.