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One-step approximations for detecting regime changes in the state space model with application to the influenza data

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Abstract

Kim and Nelson [1999. State Space Models with Regime Switching. MIT Press, Cambridge, MA] and others extended the framework of state space models involving independent regime changes to the Markov dependent case. The cost of dealing with state space models with Markov switching is high in computational effort because of the number of the possible paths through the chain. Thus it is necessary to make some approximations in order to obtain a computationally feasible algorithm for estimation. The approximations depend on modified smoothing and filtering recursions that can be easily incorporated into an EM algorithm for maximum likelihood estimation. To investigate the accuracy of approximations, we develop a new method to obtain more exact solutions, and then compare the two methods. We apply both methods to a simulated series. The result shows that employing the approximation-based algorithm not only provides accurate results but also leads to a significant reduction in the computational costs. We also apply the methods to an influenza mortality series, in which we develop a model that is general enough to include most structural models useful in monitoring changes of regime. The model proposed has the flexibility to deal with a wide range of problems involving possible regime shifts in pattern that may be seen to occur in many biological, medical and epidemiological studies.

Keywords: Hidden Markov model; State space model with switching; Kalman filter; Kalman smoother; Markov chain Monte Carlo; Gibbs sampling; EM algorithm; Influenza

1. Introduction

The state space model has become a powerful tool for modeling and forecasting dynamic systems. A growing number of published papers that employ it demonstrate its usefulness and broadness of application. In essence, a state space model is one in which an observed variable is the sum of a linear function of the state variable and an error. The state variable, in turn, evolves according to a stochastic difference equation that depends on parameters that are generally unknown. Hence, for such a state space model, given the observed data, estimations of the unobserved state vector and the parameters are of primary interest.

A switching state space model is obtained, if we assume that, in addition to the unobserved state variable, a discrete unknown switching variable influences the distribution of the observed data. A typical way of including a switching

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mechanism into a Gaussian state space model is to assume that one of the variances, e.g. the variance appearing in the observation equation is heteroskedastic and switches between various values depending on the state of the switching variable (Pena and Guttman, 1988). Alternatively, one or all the variances of the transition equation may be Markov switching heteroskedastic (Kim, 1993; Engle and Kim, 1999). Another way of including a switching mechanism is to assume that a drift term is present in the transition equation which switches between various values (Kim, 1994). Shumway and Stoffer (1991) considered the measurement matrices as switching endogenously according to an independent random process. They assumed that the possible configurations are state in a nonstationary independent process defined by the time-varying probabilities independent of past measurement matrices and of past data. In this paper, we consider the state space model with Markov switching. This means that the measurement matrices switch according to states in a hidden Markov chain.

Compared to the state space model with independent switching, the difficulty in extending the filtering algorithm to the state space model with Markov switching is the dependence among the data and the fact that the Markov switching process, s_t, is unobserved, which makes it necessary to enumerate over all possible histories to derive the filtering equations. The computation involved, requiring integration over mixtures of normal distributions, is excessively complicated without some approximations. In Section 2 we consider an approximate EM (AEM) algorithm for model (1) below.

Our primary objective of this paper is to evaluate the effects of the approximation. To this end, we develop a Monte Carlo type of EM (MCEM) to obtain more accurate solutions for the smoothed states or state probabilities in Section 3. In Section 4, we apply the AEM and MCEM methods to a simulated series and conclude with an application of a switching state space model to analyze the monthly pneumonia and influenza deaths in the US from 1968 to 1978. We focus on the following two objectives. The first objective is to compare the smoother obtained using the approximations to those using Markov Chain Monte Carlo. A second objective is to compare the parameter estimates obtained using the AEM and MCEM procedures.

2. The approximate EM (AEM)

2.1. The one-step approximation

We consider the following state space model with Markov switching:

$$x_t = \phi x_{t-1} + w_t,$$

$$y_t = A_{s_t} x_t + v_t, \quad t = 1, \ldots, n. \tag{1}$$

The state equation $x_t = \phi x_{t-1} + w_t$ models the evolution of $p \times 1$ state vector x_t from the past state vector x_{t-1}, $t = 1, 2, \ldots, n$. We assume that the w_t are $p \times 1$ independent and identically distributed, zero-mean normal vectors with covariance matrix Q. We also assume that the process starts with a normal vector x_0 that has mean μ_0 and $p \times p$ covariance matrix Σ_0. In this model, we do not observe the state vectors x_t directly, but can only observe a linear transformed version of it y_t. The measurement matrices A_{s_t} are dependent on an unobserved, discrete-valued, M-state Markov-switching variable s_t with initial probabilities $\pi_j = P(s_1 = j)$ and transition probabilities $\pi_{ij} = P(s_t = j | s_{t-1} = i)$, $i, j = 1, 2, 3, \ldots, M$, where $\sum_{j=1}^{M} \pi_{ij} = 1$ for all i. Hence, A_{s_t} is random with probability of assuming any one of M possible values $A_1, \ldots, A_M$ at any time point t. The error vectors v_t are independent zero-mean white noise vectors with common covariance R.

Denote the observed vector up to time t as $Y_t = [y_1, y_2, \ldots, y_t]$. The treatment of the state space model given by (1) depends upon being able to compute a forecast of x_t which is formed not only based on Y_{t-1}, but also conditional on the random variable s_t taking on the value j. Throughout this paper, we will use the following definitions:

$$x_{t|s}^{(j)} = E(x_t | Y_s, s_t = j), \quad x_{t|s} = E(x_t | Y_s),$$

$$P_{t|s}^{(j)} = E((x_t - x_{t|s}^{(j)})(x_t - x_{t|s}^{(j)})' | Y_s, s_t = j),$$

and

$$P_{t|s} = E((x_t - x_{t|s})(x_t - x_{t|s})' | Y_s).$$