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A new approach of goodness-of-fit testing for exponentiated laws applied to the generalized Rayleigh distribution

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Abstract

Goodness-of-fit statistics are considered which are appropriate for generalized families of distributions, resulting from exponentiation. The tests employ a variation of the data determined by the cumulative distribution function of the corresponding non-generalized distribution. The resulting test, which makes use of the Mellin transform of the transformed data, is shown to be consistent. Simulation results for the case of the generalized Rayleigh distribution show that the proposed test compares well with standard methods based on the empirical distribution function.

Keywords: Exponentiated distribution; Parametric bootstrap; Empirical Mellin transform

1. Introduction

In recent years, several standard life-time distributions have been generalized via exponentiation. Examples of such exponentiated distributions (ED) are the exponentiated Weibull family, the exponentiated exponential distribution, the exponentiated (or generalized) Rayleigh distribution, and the exponentiated Pareto family of distributions. Amongst the authors who have considered ED are, for instance, Mudholkar and Hutson (1996), Gupta and Kundu (2001), Surles and Padgett (2001), and Kundu and Gupta (2007a, b). A common feature in families of ED is that the distribution function (DF) may be written as $F(x)=[G(x)]^{\alpha}$, where $G(\cdot)$ is the DF of a corresponding non-generalized distribution, and $\alpha > 0$ denotes the generalizing parameter. Recall that the Mellin transform is defined as $\int_0^{\infty} x^{t-1} f(x)\,\mathrm{d}x$, and consequently observe that if a random variable $X > 0$ follows an ED, then the (modified) Mellin transform $M(t)=\mathbf{E}(Y^t)$ of $Y=G(X)$, exists for all $t > 0$. In particular,

$$M(t) = \frac{\alpha}{\alpha + t} \quad \forall t > 0. \tag{1.1}$$

In this paper we construct tests which rely on the empirical counterpart of $M(t)$, resulting from the observations.

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Specifically, we consider the goodness-of-fit problem of testing the null hypothesis

$$H_0 : \text{The law of } X \text{ is } [G(x;\vartheta)]^\alpha \text{ for some fixed } G(\cdot),$$

where $\{X_j\}_{j=1}^n$ denote independent observations on the random variable $X \geqslant 0$, and α, ϑ, denote unknown parameters. Notice that under H_0, (1.1) implies that the MT of $Y = G(X;\vartheta)$, satisfies, $D(\alpha;t) = 0$, $\forall t > 0$, where

$$D(\alpha;t) = (\alpha + t)M(t) - \alpha. \tag{1.2}$$

Hence we may test H_0, by employing the data

$$\hat{Y}_j = G(X_j, \hat{\vartheta}_n), \quad j = 1, 2, \ldots, n, \tag{1.3}$$

where $\hat{\vartheta}_n$ denotes a consistent estimator of the parameter ϑ. In view of (1.1) and by means of the transformation (1.3), it is intuitively possible to base a test for H_0 on a measure of deviation from zero of the random function $D_n(t) = (\hat{\alpha}_n + t)M_n(t) - \hat{\alpha}_n$ on $[0, \infty]$, where

$$M_n(t) = \frac{1}{n}\sum_{j=1}^n \hat{Y}_j^t$$

is the empirical MT of the data $\{\hat{Y}_j\}_{j=1}^n$, and $\hat{\alpha}_n$ denotes a consistent estimator of α, such as the moment estimator or the maximum likelihood estimator. In particular we propose to reject H_0 for large values of the statistic,

$$T_n = n\int_0^\infty D_n^2(t)w(t)\,\mathrm{d}t, \tag{1.4}$$

where $w(t)$ denotes a non-negative weight function, which renders the integral in (1.4) finite. Although other empirical transforms appear frequently in the literature (some of the most recent references are, Klar and Meintanis, 2005; Klar, 2005, and Cabaña and Quiroz, 2005), this is to the best of our knowledge the first time that statistical inference is performed via the empirical MT.

The rest of the paper is organized as follows. In Section 2 we study the consistency of the test statistic against general alternatives, and specify conditions which render the test statistic in a convenient form for computer implementation. The special case of the generalized Rayleigh distribution is considered in Section 3, where the new test is compared to classical goodness-of-fit tests via Monte Carlo. We conclude in Section 4 where some real data applications may also be found.

2. Consistency and specification of the test statistic

Here and in what follows, the notation $\to$ means almost sure convergence, and U(0, 1) denotes the uniform distribution in the interval (0, 1). For two stochastic sequences, we write $u_n = \mathrm{O}(v_n)$ when u_n/v_n is almost surely bounded, and $u_n = \mathrm{o}(v_n)$ when u_n/v_n is almost surely negligible, as $n \to \infty$.

In order to study the behavior of T_n under alternatives we require that the stochastic limits of the estimators exist, not only under H_0, but also under fixed alternatives. In particular, we assume that

$$\hat{\vartheta}_n - \vartheta_0 = \mathrm{O}(n^{-1/2}) \quad \text{and} \quad \hat{\alpha}_n - \alpha_0 = \mathrm{O}(n^{-1/2}),$$

where (ϑ_0, α_0) belongs to the same domain of definition as that of (ϑ, α) under the null hypothesis. For instance, when ϑ denotes scale and since α is always positive, we assume that $(\vartheta_0, \alpha_0) \in \mathbb{R}^+ \times \mathbb{R}^+$, not only under H_0, but also under the alternative hypothesis. The presentation here is for scalar ϑ, but can be extended to a vector-parameter in a straightforward manner.