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Cox proportional hazards models with frailty for negatively correlated employment processes

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ABSTRACT

In promotion discrimination cases, individuals affected by discrimination may decide to retire earlier than otherwise. Two Cox proportional hazards models are used to describe the promotion process from non-retired employees and the retirement process, respectively. To account for a potential negative correlation between the two outcomes, promotion and retirement, frailty terms are introduced. Model diagnoses in the presence of unobserved frailty terms are difficult. Therefore, the robustness of the parameter estimates to the fitting of an unnecessary frailty or a frailty distribution or form different from the underlying one is examined. The data from a reverse discrimination case, *Alexander v. Milwaukee*, are analyzed. The original finding of liability relying on a statistically significant coefficient for membership in the legally protected group (White-male) is shown to be robust to several choices of the frailty model. This provides further support for the court's decision.
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1. Introduction

Correlated time-to-event data arise frequently in a wide variety of disciplines, such as clinical trials, epidemiological studies, industrial quality control and sociology research. In this manuscript, correlated employment processes are studied in the context of equal employment opportunity cases. Seventeen White-male lieutenants brought a reverse discrimination case concerning the promotion to captain against the police department of the City of Milwaukee (*Alexander v. Milwaukee*, 474 F. 3d 437, 7th Cir. 2007). During the tenure of the police Chief (1996–2003), 41 promotions (21 White-males and 20 non-White-males) were made from the pool of 112 White-male and 34 non-White-male lieutenants who were eligible for promotion sometime during the seven-year period. Only 19% (21/112) of the White-male lieutenants were promoted compared to 59% (20/34) in the non-White-male group. Furthermore, the average duration in the lieutenant rank of the promoted White-male lieutenants was 7.36 years (the standard deviation, minimum, 25th quantile, median, 75th quantile and maximum are 2.99, 2.53, 5.48, 6.57, 10.14 and 14.78 years, respectively), while the promoted non-White-males only served an average of 3.02 years as lieutenants (the standard deviation, minimum, 25th quantile, median, 75th quantile and maximum are 2.40, 0.54, 1.15, 1.64, 3.87 and 8.41 years, respectively). In the same period, 45 out of 64 White-male officers who became eligible for retirement with pension took this option, while three of the six eligible non-White-male lieutenants retired.

In promotion discrimination cases, diminished chances of advancement are likely to affect employees' retirement decisions. The promotion and retirement processes are illustrated in Fig. 1 following the example of Joly et al. (2002). Similar to death and onset of a disease, which are semi-competing risks, the occurrence of retirement terminates the promotion process, but not vice versa. In the medical context, Xu et al. (2010) modeled semi-competing risks by three

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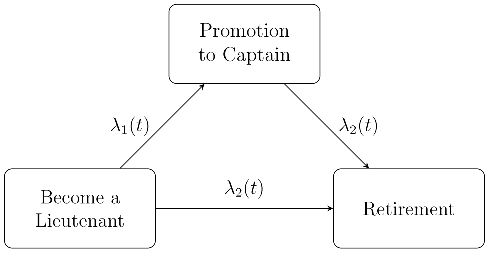

Fig. 1. $\lambda_1(t)$ and $\lambda_2(t)$ are the hazard functions of the promotion process before retirement and the retirement process.

sub-processes: the disease process before death, the death process before disease occurrence and the death process after the disease occurrence. Similarly, we model the promotions among non-retired employees because officers can no longer be considered for promotion after they retire. Unlike Xu et al. (2010), the retirement processes before and after promotion are not modeled separately because most officers in police departments retire shortly after they reach eligibility for pension regardless of their ranks (Goldstein, *New York Times*, 2011). In addition, the sample sizes in typical equal employment cases are limited because only data on employees or job applicants in the department during the time period when the alleged discrimination occurred are relevant. Therefore, we simplify the three sub-processes into the conditional promotion process before retirement and the retirement process.

The statistical analysis comparing the promotion and retirement risks of similarly qualified White-male and non-White-male officers plays a key role in the court's determination of liability and the subsequent calculation of economic compensation owed to prevailing plaintiffs. One concern is whether there exist unmeasured or unadjusted factors, which could be shared by the two processes and lead to biased estimates of the effects of discrimination. Such factors can be represented by frailties. O'Quigley and Stare (2002) pointed out that random effects models provide more efficient fixed effects estimators compared to stratified Cox models. The asymptotic properties of the maximum profile likelihood estimators from Cox proportional hazard models with frailties were provided in Nielsen et al. (1992). Klein and Moeschberger (1997) gave the EM algorithm fitting proportional hazards models with a Gamma frailty multiplying both hazard functions, where the conditional expectations of the frailty functions have explicit expressions. In situations where conditional expectations in the E step do not have closed forms, either the Monte Carlo method (Clayton, 1991) or the Gaussian quadrature algorithm (Liu and Huang, 2008) can be employed. Duchateau and Janssen (2008) discussed different parametric and semi-parametric models using frailty terms that capture positive correlations within a cluster. A unique aspect of the *Alexander v. Milwaukee* data is that employees are more likely to retire when they either believe or feel that their opportunity for further advancement is limited, suggesting that the conditional promotion and retirement processes are likely to be negatively correlated. Liu et al. (2004) proposed two Cox models for the intensity function of recurrent events and the hazard function of death, where the hazard function is multiplied by a frailty and the intensity function is multiplied by a function of the frailty term. Therefore, their model accommodates either positive or negative correlation, whose sign is estimated from the data. Liu and Yu (2008) showed how a wide family of frailty distributions can be utilized in the Liu et al. (2004) model, on which our analysis is based.

Frailty models assume that the existence, distribution and functional form of the frailty terms are known. However, it is difficult to check these assumptions (Heckman and Singer, 1984). Commenges and Andersen (1995) proposed a score test for the existence of frailty terms. However, the power of the test is low when either the sample size or the variance of the frailty is small. Therefore it is useful to know whether inferences on the effects of primary interest are robust or sensitive to misspecified models, e.g., fitting marginal models in the presence of frailties, or fitting frailty models in the absence of frailties, or choosing a simpler or more complicated function of the frailty term, or choosing a distribution other than the underlying one. The focus of this manuscript is on the robustness of the estimators of the regression coefficients to the choice of frailty distribution and functional form.

In Section 2, the models along with the estimation algorithm are presented. In Section 3, extensive simulations examining the performance of the estimators under various situations are carried out. In Section 4, appropriate models are chosen and fitted to the data from the *Alexander v. Milwaukee* case. The results provide strong support for the original finding of liability. Section 5 summarizes our findings and discusses the implications of our analysis and some remaining research problems.

2. Methods

2.1. Models

Let C_i, T_{1i}^* and T_{2i}^* be the censoring time, retirement time and promotion time for the ith subject ($i = 1, 2, \ldots, n$), respectively. The observed times for the retirement and promotion processes are $T_{1i} = \min(C_i, T_{1i}^*)$ and $T_{2i} = \min(C_i, T_{1i}^*, T_{2i}^*)$. The corresponding event indicators are $\delta_{1i} = I(T_{1i} = T_{1i}^*)$ and $\delta_{2i} = I(T_{2i} = T_{2i}^*)$, where $I(A) = 1$ if A is true and $I(A) = 0$ otherwise. In addition to the censoring that occurs where $T_{1i}^* > C_i$ or $T_{2i}^* > C_i$, the promotion process is terminated by the retirement process when $T_{1i}^* < T_{2i}^*$. When this occurs, $\delta_{2i} = 0$ and $T_{2i} = \min(T_{1i}^*, C_i)$. The numbers of events by time

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