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Establishing strong connectivity using optimal radius half-disk antennas

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A B S T R A C T

Given a set S of points in the plane representing wireless devices, each point equipped with a directional antenna of radius r and aperture angle $\alpha \geqslant 180°$, our goal is to find orientations and a minimum r for these antennas such that the induced communication graph is strongly connected. We show that $r = \sqrt{3}$ if $\alpha \in [180°, 240°)$, $r = \sqrt{2}$ if $\alpha \in [240°, 270°)$, $r = 2\sin(36°)$ if $\alpha \in [270°, 288°)$, and $r = 1$ if $\alpha \geqslant 288°$ suffices to establish strong connectivity, assuming that the longest edge in the Euclidean minimum spanning tree of S is 1. These results are worst-case optimal and match the lower bounds presented in [I. Caragiannis, C. Kaklamanis, E. Kranakis, D. Krizanc, A. Wiese, Communication in wireless networks with directional antennae, in: Proc. of the 20th Symp. on Parallelism in Algorithms and Architectures, 2008, pp. 344–351]. In contrast, $r = 2$ is sometimes necessary when $\alpha < 180°$.

1. Introduction

Consider a wireless network modeled by a set of planar point sites S, each equipped with a transceiver having a transmission radius r. Typically one assumes that communication is omni-directional and two nodes can directly communicate with each other if the distance separating them is r or less. Geometrically, the transmission region of an antenna at a point p is modeled by a circle of radius r centered at p. The connectivity of the network can be represented by a communication graph $G(S)$, which has a node for each point and an edge between each pair of nodes separated by distance r or less.

Recently there has been interest in using directional antennas in place of their omni-directional counterparts [3,5–8,10,11]. Some advantages of using directional antennas are enhanced security and reduced communication interference. Furthermore, if directional antennas are cleverly used, the power consumption of the network may be reduced. The transmission region of a directional antenna at a node p is geometrically represented by the sector of a circle with its apex at p, a central

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Table 1
Communication radius needed to achieve connectivity as a function of the aperture angle α (for $\alpha \geqslant 180°$).

Aperture	$[180°, 240°)$	$[240°, 270°)$	$[270°, 288°)$	$[288°, 360°]$
Bound	$\sqrt{3}$	$\sqrt{2}$	$2\sin 36° \approx 1.1756$	1

angle α, and a radius r. Its orientation is determined by a rotation θ about p. We assume that all antennas have the same α and r; it is only θ that varies. Thus communication between two nodes is no longer symmetric and is best modeled by a directed communication graph in which a directed edge $\overrightarrow{pq}$ indicates that q lies in p's sector.

The *direction assignment problem* is the task of finding orientations for a set of directional antennas such that the induced communication graph has certain desired properties. In this paper we focus on obtaining a strongly connected communication graph using minimal r. We will assume that S is normalized so that the length of the longest edge in its Euclidean minimum spanning tree is 1 (otherwise, we can scale the problem instance so that this property is satisfied). It is easy to see that to achieve connectivity in the normalized point set, r must be at least 1. Non-trivial lower bounds between the minimum communication radius and α were given by Caragiannis et al. [5] (the exact bounds can be seen in Table 1). These bounds correspond to the case in which there is one node p located at the origin and k other nodes forming a regular k-gon inscribed in the unit circle centered at p (for $k = 3, 4, 5$ and 6).

In this paper, we show that the lower bounds of Caragiannis et al. are tight for any $\alpha \geqslant 180°$. Our proofs are constructive. That is, we design algorithms, for any $\alpha \geqslant 180°$, that orient the antennas at a given set of nodes so as to construct a strongly connected network in which no antenna is assigned a communication radius larger than the corresponding bound listed in Table 1.

In addition to providing lower bounds on r, Caragiannis et al. [5] also gave an algorithm for orienting antennas with $180° \leqslant \alpha < 288°$ to obtain strong connectivity using $r = 2\sin(180° - \alpha/2)$ (notice that when $\alpha \geqslant 288°$ the problem is trivial). Thus, the algorithms presented here strictly improve their methods for the interval $\alpha \in [180°, 288°)$. Damian and Flatland [8] consider directional antenna of fixed aperture angles of 120° and 90°, and provide bounds of $r = 5$ and $r = 7$ (resp.) while at the same time bounding the number of hops to 5 and 6 (resp.) for nodes within unit distance. Bose et al. [11] have recently shown that a connected network using omni-directional communication can be replaced with directional antennas (with any $\alpha > 0°$) so that the increase of r and hop distance are bounded by constant factors (which depend on α). In particular, they show that for $\alpha < 60°$ and $k = 360°/\alpha$, a radius of $4\sqrt{2}(3.5k - 6)$ suffices to establish a strongly connected communication network; and for $\alpha \geqslant 60°$, a radius of $4\sqrt{2}(3 + k)$ suffices to establish a strongly connected communication network. (In both cases, the network turns out to be a hop spanner of the unit disk graph as well.)

Van Nijnatten [3] considered a variant of this problem, in which a different radius is allowed for each antenna, and the goal was to minimize the overall power consumption of the network. Ben-Moshe et al. [7] also considered similar problems for 90°-antennas but restricted the orientations of the nodes to one of the four standard quadrant directions. Bhattacharya et al. [6] considered nodes with multiple directional antennas and focused on minimizing the sum of the antenna angles for a fixed r. Kranakis et al. [10] have recently published a survey of results pertaining to the use of directional antennas in wireless networks.

Results and organization The main result of this paper is the following.

Theorem 1. *For any set S of $n > 0$ points in the plane and $\alpha \geqslant 180°$, there exists an orientation of antennas of angle α at the points of S so that the communication graph is strongly connected, and the transmission radius of each antenna is at most $\sqrt{3}$ if $\alpha \in [180°, 240°)$, $\sqrt{2}$ if $\alpha \in [240°, 270°)$, $2\sin(36°)$ if $\alpha \in [270°, 288°)$, and 1 if $\alpha \geqslant 288°$. Moreover, such orientations can be found in $O(n)$ time, provided that the minimum spanning tree of S is known in advance.*

In Section 2, we present the basic algorithm for $\alpha \geqslant 180°$ and $r = \sqrt{3}$. A proof of correctness for the algorithm is given in Section 3. Finally, in Section 4 we give the necessary modifications to reduce the communication radius to the lower bounds given in Table 1 for values of α in the ranges $[240°, 270°)$, $[270°, 288°)$, and $\geqslant 288°$.

2. Orienting antennas for $\alpha = 180°$

In this section, we establish an upper bound by presenting an algorithm for orienting 180°-antennas of radius $r = \sqrt{3}$ to obtain a strongly connected communication graph. Let MST_5 be a minimum spanning tree of S with maximum degree of five, such as the one described in [1]. Our algorithm processes nodes in the order in which they are visited in a breath-first traversal of MST_5. When a node is visited, it is assigned other nodes (within distance $\sqrt{3}$) for its antenna to cover (so as to satisfy certain invariants). If a node v is assigned to cover node w, we will say that "*v points to w*", and we use the notation $v \to w$. During the traversal of MST_5, nodes are colored *white*, *gray*, or *black*. Initially all nodes are white, meaning that they have not yet been visited and do not point to any nodes. Visited nodes are black, and they point to at least one and at most two other gray or black nodes. Gray nodes are direct children of visited nodes but have not yet been visited themselves. They point to one other gray or black node.