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Unobserved component models with asymmetric conditional variances

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Abstract

Unobserved component models with GARCH disturbances are extended to allow for asymmetric responses of conditional variances to positive and negative shocks. The asymmetric conditional variance is represented by a member of the QARCH class of models. The proposed model allows to distinguish whether the possibly asymmetric conditional heteroscedasticity affects the short-run or the long-run disturbances or both. Statistical properties of the new model and the finite sample properties of a QML estimator of the parameters are analyzed. The correlogram of squared auxiliary residuals is shown to be useful to identify the conditional heteroscedasticity. Finite sample properties of squared auxiliary residuals are also analysed. Finally, the results are illustrated by fitting the model to daily series of financial and gold prices, as well as to monthly series of inflation. The behavior of volatility in both types of series is different. The conditional heteroscedasticity mainly affects the short-run component in financial prices while in the inflation series, the heteroscedasticity appears in the long-run component. Asymmetric effects are found in both types of variables.

Keywords: Auxiliary residuals; Financial series; GARCH; Inflation; Leverage effect; QARCH; Structural time series models

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1. Introduction

Economic time series can often be decomposed into components that have a direct interpretation, for example, trend, seasonal and transitory components; see Harvey (1989) for a detailed description of unobserved component models. In the simplest case, the series of interest, y_t, can be decomposed in a long-run component, representing an evolving level, μ_t, and a transitory component, ε_t. If the level follows a random walk and the transitory component is white noise, the resulting model is given by

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, \\ \mu_t &= \mu_{t-1} + \eta_t, \end{aligned} \tag{1}$$

where ε_t and η_t are mutually independent Gaussian white noise processes with variances h and q respectively. Model (1) is known as random walk plus noise and has been very useful to represent the dynamic dependence of a large number of economic time series; see, for example, Durbin and Koopman (2001) for several applications concerning this model.

The random walk plus noise model in (1) was extended by Harvey et al. (1992) to allow the variances of both, the short- and the long-run components, to evolve over time following GARCH(1,1) models. In particular, the disturbances are defined by $\varepsilon_t = \varepsilon_t^{\dagger} h_t^{1/2}$ and $\eta_t = \eta_t^{\dagger} q_t^{1/2}$ where $\varepsilon_t^{\dagger}$ and $\eta_t^{\dagger}$ are mutually independent Gaussian white noise processes and h_t and q_t are given by

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}, \\ q_t &= \gamma_0 + \gamma_1 \eta_{t-1}^2 + \gamma_2 q_{t-1}, \end{aligned} \tag{2}$$

where the parameters α_0, α_1, α_2, γ_0, γ_1 and γ_2 satisfy the usual conditions to guarantee the positivity and stationarity of h_t and q_t.

Model (1) with the variances defined as in (2) is a Structural ARCH (STARCH) model. The main attractive of STARCH models is that they are able to distinguish whether the ARCH effects appear in the permanent and/or in the transitory component. An alternative heteroscedastic unobserved component model is proposed by Ord et al. (1997), where, instead of considering different disturbances for each component, the source of randomness is unique.

Unobserved component models with GARCH disturbances have been applied in fields like macroeconomics and finance. For example, Evans and Wachtel (1993), Ball and Cecchetti (1990) and Evans (1991) analyze inflation, Kim (1993) analyzes inflation and interest rates, Fiorentini and Maravall (1996) analyze the Spanish money supply and Diebold and Nerlove (1989), King et al. (1994), Morgan and Trevor (1999), Hasbrouck (1999), Bos et al. (2000) and Wei (2002) have applications to financial data.

The variances in Eqs. (2) are specified in such a way that their responses to positive and negative changes in the corresponding disturbances are symmetric. However, in some cases, the empirical evidence suggest that the conditional variance may have a different response to shocks of the same magnitude but different sign. This phenomenon, known as "*leverage effect*" in the Financial Econometrics literature, has often been observed in high frequency financial data; see, for example, Shephard (1996) and the references therein. In the