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An iterated parametric approach to nonstationary signal extraction

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Abstract

Consider the three-component time series model that decomposes observed data (Y) into the sum of seasonal (S), trend (T), and irregular (I) portions. Assuming that S and T are nonstationary and that I is stationary, it is demonstrated that widely used Wiener–Kolmogorov signal extraction estimates of S and T can be obtained through an iteration scheme applied to optimal estimates derived from reduced two-component models for S plus I and T plus I. This "bootstrapping" signal extraction methodology is reminiscent of the iterated nonparametric approach of the US Census Bureau's X-11 program. The analysis of the iteration scheme provides insight into the algebraic relationship between full model and reduced model signal extraction estimates.
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Keywords: ARIMA component model; Nonstationary time series; Seasonal adjustment; Signal extraction; Wiener–Kolmogorov filtering; X-11

1. Introduction

Extraction of a nonstationary signal from an observed, finite sample time series is a problem of both theoretical and practical interest. Work on stationary signal extraction from a signal plus noise model for an infinite sample dates back to Wiener (1949) and Kolmogorov (1939, 1941), whose celebrated solution has become classical in the time series literature. However, in many realistic situations, such as the project of deseasonalizing

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economic data, the ambient signal is a nonstationary stochastic process. Essentially the same Wiener–Kolmogorov filter gives optimal extractions when the signal can be made stationary by appropriate differencing, and the noise is stationary—Cleveland and Tiao (1976) obtained results for this case. Also see Hannan (1967) and Sobel (1967) for earlier work. However, when the noise process is itself nonstationary—for example, in seasonal component estimation the noise consists of trend plus irregular—the situation becomes more complicated. Bell (1984) brought this issue to the forefront with an important paper, which produced extraction estimates under a variety of assumptions; in particular, Bell demonstrated that in order to obtain optimal estimates (in the sense of mean-squared error), it was essential to make assumptions about the data-generation process, such as Assumption A.

Assumption A states that the initial observed values are probabilistically independent of the differenced signal and differenced noise processes. When the differencing operators for the signal and noise components have no unit roots in common, Assumption A has the consequence that the initial values are independent of forecasts made from the differenced data—an assumption that is often made in the modelling and forecasting of time series. Assumption A is the approach that is implicitly adopted in the literature. One of the reasons is that given above—namely, future values of the differenced series are independent of the initial observed data; a second reason is that the formulas for the optimal signal extraction estimates are much simpler analytically—Bell (1984) shows that these formulas are analogous to those used in the stationary components scenario. A third appeal of Assumption A is that we are not required to know the covariance matrix of the initial conditions of the nonstationary process; fourthly, signal extraction estimates obtained under Assumption A are also locally optimal when Assumption A is removed—namely, those signal extractions are optimal (in the sense of having minimal mean squared error) within the class of linear functions of the data such that the error in the estimate does not depend on the initial values. This property underlies the "transformation approach" of Ansley and Kohn (1985), and is appealing because no assumptions need be made on the data-generation process. See also Kohn and Ansley (1986, 1987), Bell and Hilmer (1991), De Jong (1991), Koopman (1997), and Durbin and Koopman (2001) for implementations of the Kalman filter and smoother to produce estimates that are optimal under Assumption A.

In a three component model—consisting of trend, seasonal, and irregular portions—used to describe economic data, quite often the trend and seasonal are modelled as nonstationary processes, whereas the irregular is stationary. If one is interested in obtaining the trend, one must use signal extraction methods for a nonstationary signal (the trend) plus a nonstationary noise (the seasonal plus irregular) component. Under Assumption A, the finite-sample matrix formulas of Bell and Hilmer (1988) and Bell (2004) can be used; equivalently, a state space smoother (see Anderson and Moore, 1979) can produce the trend estimate once a model has been specified for each component. Of course, other components like a cycle can be handled by the Kalman smoother.

Another approach is to first detrend the data, by using trend extraction methods for a reduced "trend plus irregular" model; although this is an inaccurate depiction of reality, the matrix form of this estimate is easy to write down, since the noise (i.e., the irregular) is now stationary. After subtracting off this pilot trend estimate, one can then extract the seasonal component using a reduced "seasonal plus irregular" model—again, this model is not true