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Bootstrap prediction for returns and volatilities in GARCH models

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Abstract

A new bootstrap procedure to obtain prediction densities of returns and volatilities of GARCH processes is proposed. Financial market participants have shown an increasing interest in prediction intervals as measures of uncertainty. Furthermore, accurate predictions of volatilities are critical for many financial models. The advantages of the proposed method are that it allows incorporation of parameter uncertainty and does not rely on distributional assumptions. The finite sample properties are analyzed by an extensive Monte Carlo simulation. Finally, the technique is applied to the Madrid Stock Market index, IBEX-35.

Keywords: Time series; Non-Gaussian distributions; Nonlinear models; Resampling methods

1. Introduction

Financial market participants have an increasing interest in prediction intervals for future returns as measures of uncertainty. For example, in the area of financial risk management, it is important to provide density forecasts of portfolio prices and to track certain aspects of these densities such as value at risk (VaR); see, for example, Bollerslev (2001) and Engle (2001). On the other hand, accurate predictions of future volatilities are critical for the implementation and evaluation of asset and derivative pricing theories as well as trading and

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hedging strategies. It is by now well documented in the literature that the volatility of financial returns evolves over time. Generalized autoregressive conditionally heteroscedastic (GARCH) models, originally introduced by Bollerslev (1986), provide dynamic prediction intervals which are narrow in quiet times and wide in volatile periods. However, despite the extensive literature related with GARCH models, relatively little attention has been given to the construction of prediction intervals for GARCH models. Furthermore, the literature on volatility prediction has traditionally dealt with point forecasts without considering any measure of the uncertainty associated to forecasting future volatilities. The interest of having this measure has been put forward, among many others, by Baillie and Bollerslev (1992), Shephard (1996), Andersen and Bollerslev (1998), Andersen et al. (2001), Engle and Patton (2001) and Tsay (2002). The few procedures available in the literature to obtain prediction intervals for future volatilities are mainly Bayesian and need to assume a particular distribution for the errors; see, for example, Jacquier et al. (1994).

In this paper, we propose to use bootstrap methods to obtain prediction densities of future returns and volatilities generated by GARCH models. Our proposal is a generalization of the procedure proposed by Pascual et al. (2004) for linear autoregressive integrated moving average (ARIMA) models. The resulting prediction intervals for returns and volatilities incorporate the uncertainty due to parameter estimation without distributional assumptions on the sequence of innovations. Miguel and Olave (1999) have also proposed a bootstrap procedure to obtain prediction intervals for future observations generated by ARCH models but their intervals do not incorporate the parameter uncertainty. Consequently, their proposal does not allow to construct prediction intervals for one-step ahead future volatilities.

The paper is organized as follows. Section 2 describes the main properties of GARCH processes and predictions. In Section 3, we present the proposed resampling procedure to estimate prediction densities and intervals for returns and volatilities. Its finite sample behavior is analyzed in Section 4, which reports the results of an extensive Monte Carlo simulation study. Section 5 presents an application with real financial data. Finally, the conclusions appear in Section 6.

2. The GARCH(1,1) model

The GARCH(1,1) model provides a simple representation of the main dynamic characteristics of returns of a wide range of assets and it is extensively used to model them. Hence, although it could not be the optimal model for volatility forecasting in any given series, it serves as a natural benchmark for the forecast performance of heteroscedastic models. For simplicity, we concentrate on it hereafter. However, the bootstrap procedure proposed in the following section can be directly generalized to general GARCH(p,q) processes. A GARCH(1,1) model is given by

$$y_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \quad t = 1, \ldots, T, \tag{1}$$

where ε_t is a white noise process with unit variance, σ_t is a stochastic process known as volatility and assumed to be independent of ε_t, and ω, α and β are unknown parameters that