Computational Statistics and Data Analysis 60 (2013) 97–110

Contents lists available at SciVerse ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

A Bayesian semiparametric model for volatility with a leverage effect

E.-I. Delatola, J.E. Griffin *

School of Mathematics, Statistics and Actuarial Science, University of Kent, Canterbury, CT2 7NF, UK

ARTICLE INFO

Article history:
Received 22 November 2011
Received in revised form 23 October 2012
Accepted 30 October 2012
Available online 16 November 2012

Keywords:
Dirichlet process
Asset return
Stock index
Off-set mixture representation
Mixture model
Centred representation

ABSTRACT

A Bayesian semiparametric stochastic volatility model for financial data is developed. This nonparametrically estimates the return distribution from the data allowing for stylized facts such as heavy tails of the distribution of returns whilst also allowing for correlation between the returns and changes in volatility, which is usually termed the leverage effect. An efficient MCMC algorithm is described for inference. The model is applied to simulated data and two real data sets. The results of fitting the model to these data show that choosing a parametric return distribution can have a substantial effect on inference about the leverage effect.

1. Introduction

In the last couple of decades, stochastic volatility (SV) models have enjoyed great popularity for analysing financial data. This popularity can be mainly attributed to the development of new, more advanced techniques in econometrics, as well as the availability of rapidly increasing computing power. The SV model as introduced by Taylor (1982) captured the heterogeneity of daily returns of sugar prices using a latent autoregressive process of order 1 for the logged variance of a normal return distribution. This model allowed heavy-tails for the unconditional distribution of returns and time-varying volatility. However, this model was unable to capture other features of financial data such as heavy tails of the conditional distribution of returns, price jumps and the leverage effect. Black (1976) introduced the term leverage effect when observing that a positive return tends to lead to a smaller increase in its conditional variance than a negative return of the same size. Following the work of Taylor (1982), many extensions of the SV model have been introduced incorporating such stylized features. This paper will address the issue of building an SV model with a leverage effect and a heavy tailed conditional distribution of returns within the Bayesian nonparametric framework.

Harvey and Shephard (1996) introduced an SV model that could capture the leverage effect. Let P_t denote the daily price of an asset or a stock index at time t for $t = 1, \ldots, n$. The daily return of the asset or the stock index at time t is defined to be $y_t = \frac{P_t}{P_{t-1}} - 1$. The nonlinear SV model with leverage of Harvey and Shephard (1996) is

$$y_t = \beta \exp(h_t/2)\, \epsilon_t \tag{1}$$

and

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t$$

where $\beta > 0$, h_t is the log-volatility at time t and ϕ is a persistence parameter, for which it is assumed that $|\phi| \leq 1$ to ensure the stationarity of h_t. The parameters β and μ both control the unconditional variance of y_t and the model is usually made

* Corresponding author. Tel.: +44 2476 523369; fax: +44 2476 524532.
E-mail addresses: ed62@kentforlife.net (E.-I. Delatola), J.E.Griffin-28@kent.ac.uk (J.E. Griffin).

0167-9473/$ – see front matter
doi:10.1016/j.csda.2012.10.023

identifiable by assuming that either $\beta = 1$ or $\mu = 0$. Unlike earlier SV models, the error terms (ϵ_t, η_t) are independently and identically distributed according to a bivariate normal distribution with mean $\mathbf{0} = (0, 0)'$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{pmatrix}$$

where σ_η^2 is the variance of the increment η_t of the log-volatility and ρ is the correlation between the error terms. The addition of this parameter introduces correlation between the errors in the return distribution ϵ_t and the increments in the log-volatility from time t to time $t+1$ and so allows the model to capture the leverage effect. The volatility at time $t = 1$ is assumed to be drawn from the stationary distribution which is $h_1 \sim N\left(\mu, \frac{\sigma_\eta^2}{1-\phi^2}\right)$, where $x \sim N(m, \sigma_x^2)$ represents that x follows a normal distribution with mean m and variance σ_x^2. Omori et al. (2007) discussed Markov chain Monte Carlo (MCMC) methods for this model. The equation for y_t non-linearly depends on h_t in (1) and so the model is a non-linear state space model. Omori et al. (2007) worked with $y_t^\star = \log(y_t + c)^2$ where c is small which leads to a linear state space model. The error terms of the return equation in this representation, $\log \epsilon_t^2$, which follow a $\log \chi_1^2$ distribution, can be accurately approximated by a 10-component mixture of normals. Nakajima and Omori (2009) extended the work of Omori et al. (2007) to incorporate jumps and heavy tails. An extension of the stochastic volatility to include leverage and heavy tails was also proposed by Jacquier et al. (2004) who make posterior inference using the non-linear representation of the model.

Several Bayesian nonparametric approaches to modelling the heavy tails in financial time series have recently been proposed. Ausín et al. (2010) and Kalli et al. (2011) introduced GARCH semiparametric models. Ausín et al. (2010) suggested modelling the error terms of the return equation with a Dirichlet process mixture of normals model. They fitted their semiparametric model to both the Bombay Stock Exchange Index and the Hang Seng Index and found evidence that their model better described the tail behaviour of the return distribution. Kalli et al. (2011) introduced an alternative semiparametric GARCH model where the error terms of the return equation are modelled using an infinite mixture of scaled uniform distributions. The empirical findings are similar to those discussed in Ausín et al. (2010). In stochastic volatility models, Jensen and Maheu (2010) and Delatola and Griffin (2011) both nonparametrically modelled the return distribution using a Dirichlet process mixture of normals to define a semiparametric SV models. Jensen and Maheu (2010) used the non-linear representation of an SV model whilst Delatola and Griffin (2011) work with a linearized representation by modelling $\log(y_t^2 + c)$. Both models were shown to be better at capturing the tail behaviour of the returns than a simple SV model with a normal return distribution.

The scope of this paper is the extension of the work of Nakajima and Omori (2009) using Bayesian nonparametric techniques. The return distribution of the SV model will be flexibly modelled using a Dirichlet process mixture model which allows for several of the stylized features of returns, such as leverage and heavy tails of the conditional distribution of returns, to be captured. The flexibility of the Dirichlet process mixture model avoids the need to introduce extra parameters to capture features of the return distribution. An alternative semiparametric SV model with leverage was introduced by Jensen and Maheu (2011) who used a bivariate Dirichlet process mixture model for the errors in the nonlinear SV model with leverage. In the empirical analysis of both Jacquier et al. (2004) and Nakajima and Omori (2009), there was evidence that their SV model with heavy tails and leverage fitted the examined data better than models based on the assumption of normality. These findings show that the commonly-made assumption of normality of error terms does not hold in many cases.

The paper is structured as follows. Section 2 describes our Bayesian nonparametric model with leverage (SVL-SPM), Section 3 reviews the sampling strategy for MCMC estimation of this model, Section 4 reports applications of the method to simulated and financial data examples (Microsoft asset prices and the Standard and Poors 500 index), and Section 5 concludes.

2. Semiparametric stochastic volatility model with leverage

This section presents a flexible version of the linear state-space representation of the SV model with leverage (SVL-SPM). The SVL-SPM extends the parametric model with leverage presented by Omori et al. (2007) which will be referred to as the SVL-PM. The next two subsections summarize the concepts of the SVL-PM and the Dirichlet process mixture model respectively which will be used to build the SVL-SPM.

2.1. Parametric stochastic volatility model with leverage

A linear state-space representation of the non-linear SV model with leverage (SVL-PM) in (1) was derived by Omori et al. (2007) who take the logarithm of the squared returns. We briefly review their development. The SVL-PM is

$$y_t^\star = h_t + z_t$$

and

$$h_{t+1} = \mu + \phi\,(h_t - \mu) + \eta_t$$