Computational Statistics and Data Analysis 56 (2012) 4047–4066

Contents lists available at SciVerse ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Generalized exponential–power series distributions

Eisa Mahmoudi *, Ali Akbar Jafari

Department of Statistics, Yazd University, P.O. Box 89175-741, Yazd, Iran

ARTICLE INFO

Article history:
Received 20 January 2011
Received in revised form 20 August 2011
Accepted 15 April 2012
Available online 25 April 2012

Keywords:
EM-algorithm
Generalized exponential distribution
Maximum likelihood estimation
Power series distributions

ABSTRACT

In this paper, we introduce the generalized exponential–power series (GEPS) class of distributions, which is obtained by compounding generalized exponential and power series distributions. The compounding procedure follows the same way as previously carried out in introducing the complementary exponential–geometric (CEG) and the two-parameter Poisson–exponential (PE) lifetime distributions. This new class of distributions contains several lifetime models such as: CEG, PE, generalized exponential–binomial (GEB), generalized exponential–Poisson (GEP), generalized exponential–geometric (GEG) and generalized exponential–logarithmic (GEL) distributions as special cases.

The hazard function of the GEPS distributions can be increasing, decreasing or bathtub shaped among others. We obtain several properties of the GEPS distributions such as moments, maximum likelihood estimation procedure via an EM-algorithm and inference for a large sample. Special distributions are studied in some detail. At the end, in order to show the flexibility and potentiality of the new class of distributions, we demonstrate applications of two real data sets.

© 2012 Elsevier B.V. All rights reserved.

1. Introduction

Consider a system with N components, where N (the number of components) is a discrete random variable with support $\{1, 2, \ldots\}$. The lifetime of ith ($i = 1, 2, \ldots, N$) component is the positive continuous random variable, say X_i, whose distribution belongs to one of the lifetime distributions such as exponential, gamma, Weibull, Pareto, etc. The discrete random variable N can have some distributions such as geometric, zero-truncated Poisson, logarithmic, and the power series distributions in general. The non-negative random variable Y denoting the lifetime of such a system is defined by $Y = \min_{1\le i\le N} X_i$ or $Y = \max_{1\le i\le N} X_i$, based on whether the components are series or parallel.

In recent years, many distributions to model lifetime data have been introduced by considering a system with series components, such as the exponential–geometric (EG), exponential–Poisson (EP), exponential–logarithmic (EL), exponential–power series (EPS), Weibull–geometric (WG) and Weibull–power series (WPS) distributions which were introduced and studied by Adamidis and Loukas (1998), Kus (2007), Tahmasbi and Rezaei (2008), Chahkandi and Ganjali (2009), Barreto-Souza et al. (2011) and Morais and Barreto-Souza (2011), respectively.

By considering a system with parallel components, Barreto-Souza and Cribari-Neto (2009) and Louzada-Neto et al. (2011) introduced the exponentiated exponential–Poisson (EEP) and the complementary exponential–geometric (CEG) distributions where the EEP is the generalization of the EP distribution and the CEG is complementary to the exponential–geometric model proposed by Adamidis and Loukas (1998). Recently, Cancho et al. (2011) introduced the two-parameter Poisson–exponential lifetime distribution which arises on a latent complementary risk problem base (Basu and Klein, 1982).

By taking a system with parallel components in which the random variable N has the power series distributions and the random variable X_i follows the generalized exponential (GE) distribution, we introduce the generalized

* Corresponding author.
E-mail address: emahmoudi@yazduni.ac.ir (E. Mahmoudi).

0167-9473/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.csda.2012.04.009

exponential–power series (GEPS) class of distributions which contains the complementary exponential–geometric (CEG) and Poisson–exponential (PE) as special cases. The GEPS class of distributions contains several lifetime models such as: generalized exponential–binomial (GEB), generalized exponential–Poisson (GEP), generalized exponential–geometric (GEG) and generalized exponential–logarithmic (GEL). The main reasons for introducing the GEPS class of distributions are: (i) This class of distributions is an important model that can be used in a variety of problems in modeling lifetime data. (ii) This class of distributions is a suitable model in a complementary risk problem base in the presence of latent risks which arise in several areas such as public health, actuarial science, biomedical studies, demography and industrial reliability. (iii) It provides a reasonable parametric fit to skewed data that cannot be properly fitted by other distributions. (iv) This class contains several lifetime models as special cases.

The paper is organized as follows. In Section 2, we define the class of GEPS distributions. The density, survival and hazard rate functions and some of their properties are given in this section. In Section 3, we derive quantiles and moments of GEPS distributions. In Section 4, we present some special distributions which are studied in detail. Estimation of the parameters by maximum likelihood method and inference for large sample are presented in Section 5. The EM-algorithm with a method for evaluating the standard errors from the EM-algorithm is presented in Section 6. Simulation study is given in Section 7. Applications to two real data sets are given in Section 8. Finally Section 9 concludes the paper.

2. The class of GEPS distributions

The random variable X has a generalized exponential (GE) distribution (Gupta and Kundu, 1999) with parameters α and β if its cumulative distribution function (cdf) takes the form

$$G(x) = \left(1 - e^{-\beta x}\right)^{\alpha}, \quad x > 0,$$

where $\alpha > 0$, $\beta > 0$. The corresponding probability density function (pdf) is

$$g(x) = \alpha\beta e^{-\beta x}\left(1 - e^{-\beta x}\right)^{\alpha - 1}.$$

Given N, let $X_1, \ldots, X_N$ be independent and identically distributed (iid) random variables from GE distribution. Here, let N be a discrete random variable with a member of power series distributions (truncated at zero) with probability mass function given by

$$P(N = n) = \frac{a_n \theta^n}{C(\theta)}, \quad n = 1, 2, \ldots,$$

where $a_n \geq 0$ depends only on n, $C(\theta) = \sum_{n=1}^{\infty} a_n \theta^n$, and $\theta \in (0, s)$ is chosen in a way such that $C(\theta)$ is finite and its first, second and third derivatives are defined and shown by $C'(.)$, $C''(\cdot)$ and $C'''(\cdot)$. For more details on the power series class of distributions, see Noack (1950).

Let $X_{(n)} = \max_{1 \leq i \leq N} X_i$, then the conditional cdf of $X_{(n)}|N = n$ is given by

$$G_{X_{(n)}|N=n}(x) = (G(x))^n = \left(1 - e^{-\beta x}\right)^{n\alpha},$$

which has a GE distribution with parameters $n\alpha$ and β.

The Generalized Exponential–Power Series (GEPS) class of distributions, that we denote by GEPS(α, β, θ), is defined by the marginal cdf of $X_{(n)}$, i.e.,

$$F(x) = \sum_{n=1}^{\infty} \frac{a_n \theta^n}{C(\theta)} (G(x))^n = \frac{C(\theta G(x))}{C(\theta)} = \frac{C\left(\theta\left(1 - e^{-\beta x}\right)^{\alpha}\right)}{C(\theta)}, \quad x > 0. \tag{1}$$

This new class of distributions includes lifetime distributions presented by Cancho et al. (2011) (Poisson–exponential distribution) and Louzada-Neto et al. (2011) (complementary exponential–geometric distribution). This class also includes the two new mixtures of GE with logarithmic distribution (GEL) and Binomial distribution (GEB).

Remark 1. Let $X_{(1)} = \min_{1 \leq i \leq N} X_i$, then the cdf of $X_{(1)}$ is

$$F_{X_{(1)}}(x) = 1 - \frac{C(\theta - \theta G(x))}{C(\theta)} = 1 - \frac{C\left(\theta - \theta\left(1 - e^{-\beta x}\right)^{\alpha}\right)}{C(\theta)}.$$

If $\alpha = 1$, then the cdf of $X_{(1)}$ is $F_{X_{(1)}}(x) = 1 - \frac{C(e^{-\beta x})}{C(\theta)}$, which is called exponential–power series distributions (Chahkandi and Ganjali, 2009) and this family includes the lifetime distributions presented by Adamidis and Loukas (1998), Kus (2007), and Tahmasbi and Rezaei (2008).

Remark 2. Let X has $F_{X_{(1)}}(x)$ distribution. Then $Y = G^{-1}(1 - G(X))$ has GEPS distributions, where G^{-1} is the inverse function of G, because

Download English Version:

https://daneshyari.com/en/article/415796

Download Persian Version:

https://daneshyari.com/article/415796

Daneshyari.com