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CUSUM control charts for monitoring optimal portfolio weights

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ABSTRACT

A portfolio investor requires statistical tools for the timely detection of changes in the optimal portfolio composition. Several multivariate cumulative sum (CUSUM) control charts are proposed for the purpose of monitoring optimal portfolio weights. The ability of the CUSUM schemes to detect important types of changes in the optimal portfolio weights is analyzed in an extensive Monte Carlo simulation study. The empirical application of control charts shows that the proposed methodology can provide a significant reduction of the portfolio volatility.

1. Introduction

The mean-variance portfolio approach of Markowitz (1952) provides a possibility to determine optimal portfolio weights relying on the assumption of known means and covariances of the risky asset returns. Since these quantities are unknown in practice they have to be estimated from the historical data. An important benchmark in portfolio selection is the global minimum variance portfolio (GMVP), which exhibits the lowest attainable variance. Moreover, this portfolio is the starting point of the mean-variance efficient frontier. The GMVP weights depend solely on the covariance matrix of the asset returns, but not on the hardly predictable mean asset returns. The latter often cause large estimation errors in the optimal portfolio proportions (Best and Grauer, 1991). Since the covariance matrix can usually be estimated and forecasted much better, the GMVP is useful for investment decisions from both theoretical and practical viewpoints.

Since the covariances change over time, the GMVP weights alter over time as well. Assume that the GMVP weights remain unchanged over some historical period of time. Then the investor needs statistical instruments to check, whether the former GMVP composition remains optimal. That means, the investor should decide, whether the previous (null) hypothesis about the GMVP weights is still valid in the current period. Since these decisions should be made at every new point in time, this problem is of sequential nature.

Statistical process control (SPC) suggests control charts as an appropriate instrument for dealing with sequential decision problems (see, e.g. Montgomery, 2005). In SPC terminology, the investor should timely detect a non-random unknown change point, where the actual parameters of the weight process deviate from their targets, e.g. historically presumed values. A control chart consists of a control statistic and a rejection area. The chart provides a signal if the control statistic gets into the rejection area for the first time after the monitoring has started. A signal indicates that there is possibly a change in the monitored parameters, so that the null hypothesis does not hold any more. Alternatively, there could be a false signal, whereas the process parameters remain unchanged. A good chart provides quickly a signal after a change and no signal for

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a long time if no change occurs. The investor should reconsider his opinion about the current GMVP proportions after each signal.

The most popular control charts refer to exponentially weighted moving average (EWMA) (Roberts, 1959) and cumulated sum (CUSUM) (Page, 1954) families. Multivariate charts are required for monitoring the vector of the GMVP weights. Golosnoy and Schmid (2007) and Golosnoy et al. (2010) propose several multivariate EWMA control charts (cf. Lowry et al., 1992) for monitoring the GMVP composition. This paper elaborates further tools for the surveillance of the GMVP weights and contributes to the current literature in four main aspects. First, several multivariate CUSUM control charts are developed for sequentially monitoring the GMVP weights. Second, the detecting ability of the suggested CUSUM schemes is investigated within an extensive Monte Carlo simulation study. Third, we consider the possibility of combining different charts in order to exploit their individual advantages for the GMVP surveillance. Fourth, an empirical study illustrates how the information obtained by the control chart signals can be used for portfolio volatility reduction.

In the current paper we develop several CUSUM charts for monitoring changes in the GMVP composition. The monitored processes are the estimated GMVP weights $\hat{\mathbf{w}}_{n,t}$ as well as the auxiliary characteristic $\mathbf{q}_t$, which is related to the estimated GMVP weights but exhibits more appealing stochastic properties (Golosnoy et al., 2010). Thus we extend the family of EWMA schemes for the GMVP surveillance by suggesting multivariate CUSUM-w and CUSUM-q charts, respectively. The popular MCUSUM1 and MCUSUM2 charts of Pignatiello and Runger (1990) as well as the projection pursuit (PPCUSUM) scheme of Ngai and Zhang (2001) are adapted for our purposes and analyzed within a Monte Carlo study. The optimal chart design is identified for different types of changes in the covariance matrix of asset returns, leading to the alterations in the GMVP composition. Comparing the performance of the CUSUM schemes to those of the EWMA charts, we find that the CUSUM schemes exhibit a similar behavior to the EWMA charts for many types of changes in the GMVP weights. Remarkably, the CUSUM charts outperform the EWMA counterparts in detection of large shifts in the GMVP composition. Since there is no single control chart which outperforms alternative schemes for all considered shifts in the GMVP proportions, we consider the idea to combine control charts in order to exploit the advantages of the different schemes simultaneously in case of uncertainty about coming changes.

The empirical application illustrates the usefulness of control chart signals for minimizing the ex ante variance of portfolio returns. We suggest to interpret the obtained signals as a failure of the target GMVP weights and propose to close risky positions for one period in these situations. If there is another signal in the coming period, the positions stay closed, otherwise the investor selects the target portfolio again. Such a strategy leads to a significant reduction of the out-of-sample portfolio variance compared to benchmark portfolio approaches which neglect signal information. A strategy using simultaneous signals appears to be especially successful in our study.

The rest of the paper is organized as follows. Section 2 describes the portfolio problem and formulates the surveillance task for the GMVP weights. Section 3 introduces the CUSUM control charts for monitoring the processes of the GMVP weights and the auxiliary characteristics. Section 4 investigates the properties of the CUSUM charts by extensive Monte Carlo simulations. Section 5 provides an empirical application of our monitoring methodology for portfolio volatility reduction. Section 6 concludes, while the proofs are provided in the Appendix.

2. Portfolio problem

2.1. Global minimum variance portfolio

Suppose that there are k risky assets available for the portfolio composition. Assume that the vector of asset returns $\mathbf{X}_t$ follows a k-dimensional normal distribution, i.e. $\mathbf{X}_t \sim \mathcal{N}_k(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ for $t \in \mathbb{N}$. This is a common assumption for a broad class of financial models (cf. Tsay, 2005). Moreover, the asset returns are assumed to be not autocorrelated for all t, which is in line with the efficient market hypothesis. A single investment period lasts from t till $t+1$. Following Markowitz (1952), the optimal portfolio weight $\boldsymbol{w}_t$ at time t should depend on the unknown moments $\boldsymbol{\mu}_{t+1}$ and $\boldsymbol{\Sigma}_{t+1}$. In practice, these quantities have to be estimated based on the information set available at time t.

There are considerable difficulties in estimating and predicting the mean vector $\boldsymbol{\mu}_{t+1}$ (Merton, 1980). The errors arising by the mean estimation have a damaging impact on the portfolio performance (Best and Grauer, 1991). The covariance matrix $\boldsymbol{\Sigma}_{t+1}$ exhibits much more predictability because of (persistent) volatility clusters which indicate a disagreement of the market participants about future cash flows on risky assets (cf. Tsay, 2005). The volatility clusters cause pronounced positive autocorrelations in the absolute and squared series of the daily asset returns. These are the reasons for selecting the global minimum variance portfolio (GMVP), which provides the weights leading to the smallest attainable portfolio variance under restriction that the sum of the weights is equal to 1. The GMVP weights are given by

$$\mathbf{w}_t = \frac{\boldsymbol{\Sigma}_{t+1}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}_{t+1}^{-1}\mathbf{1}}, \tag{1}$$

where $\mathbf{1}$ is a k-dimensional vector whose components are equal to one. The GMVP weights do not require a knowledge of the mean vector $\boldsymbol{\mu}_{t+1}$, so that the estimation risk caused by an unknown mean returns is eliminated. Note that both negative elements as well as weights greater than 1 are permitted in the vector $\mathbf{w}_t$.