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Generalized method of moments estimation for cointegrated vector autoregressive models[☆]

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ABSTRACT

In this study, a generalized method of moments (GMM) for the estimation of nonstationary vector autoregressive models with cointegration is considered. Two iterative methods are considered: a simultaneous estimation method and a switching estimation method. The asymptotic properties of the GMM estimators of these methods are found to be the same as those of the Gaussian reduced-rank estimator. Through Monte Carlo simulation, the small-sample properties of the GMM estimators are studied and compared with those of the Gaussian reduced-rank estimator and the maximum likelihood estimator considered by other researchers. In the case of small samples, the GMM estimators are more robust to deviations from normality assumptions, particularly to outliers.

1. Introduction

The generalized method of moments (GMM) is an estimation method that helps avoid strong parametric assumptions in data analysis. After the introduction of the GMM in Hansen (1982), variants of the GMM have been used to analyze economic models in numerous fields, including finance and macroeconomics. The distributional properties of financial and macroeconomic data include skewness and leptokurtosis. Thus, in such cases, the maximum likelihood estimator (MLE), which is based on a multivariate normality assumption, may no longer have optimal properties, and hence it might provide misleading results. An advantage of the GMM is that only certain moment conditions need to be specified for the parameters to be estimated; information about the innovation distributions and autocorrelation and heteroskedasticity properties are not required.

Kitamura and Phillips (1997) developed the GMM approach for a nonstationary regression model, while Quintos (1998) extended their fully modified GMM estimators to the nonstationary regression of cointegration models. However, since Quintos (1998) was interested only in long-run relations, the effect of lagged response variables was absorbed in the innovation term. This led to correlation between the innovation term and the regressor, and instrumental variables were introduced to resolve the correlation issues. Kleibergen (1999) adopted the GMM for a simple vector error correction model (VECM), in which long-run relations were considered and short-term dynamics were neglected.

In the GMM, parameters are estimated by minimizing the objective function comprising moment conditions. Since cointegration models have a reduced rank structure among the parameters, a specific identification condition is required to

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identify the cointegrating estimator. Kleibergen (1999) obtained the first-order condition for the object function to estimate unknown parameters without employing any identification constraint, and subsequently determined the identification conditions. Consequently, calculations for the estimator became difficult, and the asymptotic distribution for the estimator was highly complex.

In this study, we assume that the cointegrating rank is known, and on the basis of the standard time series framework by Kleibergen (1999) we develop two iterative GMM estimation methods: a simultaneous estimation method and a switching estimation method. The cointegrating rank can be identified using canonical correlations in the early stages of model building, such as that described in Ahn (1997). Our GMM estimators also induce an orthogonality condition since the assumption considered in the statistical model or econometric theory is adopted, as in the case of the previous GMM. In the simultaneous estimation method, the unknown parameters are simultaneously estimated, as in Ahn and Reinsel (1990). In the switching estimation method, the unknown parameters are divided into two groups. The parameters of the first group, referred to as nonstationary parameters, are associated with the nonstationary part of the process, that is, the cointegrating vectors. The parameters of the second group, referred to as stationary parameters, are associated with the stationary part of the process. Given the second group of parameters, we can estimate the parameters of the first group, and vice versa. Unlike the estimation method proposed in Kleibergen (1999), our methods give the identification condition for the parameters in advance; then, the first-order condition of the objective function is obtained. Hence, the proposed methods involve simple calculations and are more efficient than the method of Kleibergen (1999).

The rest of this paper is organized as follows. In Section 2, we discuss our iterative GMM estimation methods for the VECM and derive the asymptotic distribution of the GMM estimators. Section 3 shows the extension of the results to models with deterministic components. In Section 4, we present the results of the Monte Carlo experiments performed using the proposed GMM estimators and the existing MLEs and discuss the extraordinary outlier problem of the MLE on the basis of the real data example of the German monetary system. Finally, concluding remarks are presented in Section 5. The proofs of the theorems stated in this paper are presented in the Appendix.

2. Two iterative GMM estimation methods

We consider an m-dimensional vector autoregreesion (VAR) process $\{\boldsymbol{y}_t\}$ given by

$$\Phi(L)\boldsymbol{y}_t = \left(I_m - \sum_{j=1}^{p} \Phi_j L^j\right)\boldsymbol{y}_t = \boldsymbol{\epsilon}_t, \tag{1}$$

where $\Phi(L) = I_m - \Phi_1 L - \cdots - \Phi_p L^p$. We assume that the characteristic equation $det(\Phi(L)) = 0$ has $d < m$ unit roots, with the remaining roots lying outside the unit circle, and that $rank(\Phi(1)) = r = m - d$. Under this assumption, $(1 - L)\boldsymbol{y}_t$ becomes a stationary process. We further assume that $\{\boldsymbol{\epsilon}_t\}$ is a sequence of independent m-dimensional random vectors with $E(\boldsymbol{\epsilon}_t) = 0$, $Cov(\boldsymbol{\epsilon}_t) = \Omega$, and $\sup_t E(|\epsilon_{j,t}|^{2+\delta}) < \infty$ for some $\delta > 0$ and $j = 1, 2, \ldots, m$.

After proper transformation, this cointegrated VAR model takes the form of the VECM; by applying the Lagrange expansion around the unit root of the polynomial $\Phi(z)$, we have the following VECM:

$$\begin{aligned}\Delta\boldsymbol{y}_t &= -\Phi(1)\boldsymbol{y}_{t-1} + \sum_{j=1}^{p-1} \Phi_j^* \Delta\boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t \\ &= \alpha\beta'\boldsymbol{y}_{t-1} + \sum_{j=1}^{p-1} \Phi_j^* \Delta\boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t,\end{aligned} \tag{2}$$

where $\Phi(1) = I_m - \sum_{j=1}^{p} \Phi_j$ with $\Phi_j^* = -\sum_{k=j+1}^{p} \Phi_k$. α and β are full-rank $(m \times r)$ matrices such that $\beta'\boldsymbol{y}_t$ describes the long-run relationships, α is an adjustment coefficient, and $\Phi_1^*, \ldots, \Phi_{p-1}^*$ represent the short-run dynamics of the process. Because model (2) considers the long-run relationships and short-run dynamics simultaneously, and the entire information for the cointegration rank can be obtained from $\Phi(1)$, the VECM is widely used for cointegration models.

For a unique identification of the cointegrating vector β, we use the same normalization $\beta' = (I_r, \beta_0{}')$ used by Ahn and Reinsel (1990), where I_r is the $r \times r$ identity matrix, and β_0 is an $(m-r) \times r$ matrix of unknown parameters. Then, the VECM can be expressed as

$$\Delta\boldsymbol{y}_t = \alpha\boldsymbol{y}_{1,t-1} + \alpha\beta_0'\boldsymbol{y}_{2,t-1} + \sum_{j=1}^{p-1} \Phi_j^* \Delta\boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{y}_{t-1} = (\boldsymbol{y}_{1,t-1}', \boldsymbol{y}_{2,t-1}{}')'$; $\boldsymbol{y}_{1,t-1}$ is an $r \times 1$ vector and $\boldsymbol{y}_{2,t-1}$ is an $(m - r) \times 1$ vector. We use the following notation to obtain the estimators easily and to describe the system in a compact form:

$$\Delta\boldsymbol{y}_t = \Pi\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_t, \tag{3}$$