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The effects of misspecified marginals and copulas on computing the value at risk: A Monte Carlo study

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Abstract

The effect on the estimation of the Value at Risk when dealing with multivariate portfolios when there is a misspecification both in the marginals and in the copulas is investigated. It is first shown that, when there is skewness in the data and symmetric marginals are used, the estimated elliptical (normal or t) copula correlations are negatively biased, reaching values as high as 70% of the true values. Besides, the bias almost doubles if negative correlations are considered, compared to positive correlations. As for the *t* copula degrees of freedom parameter, the use of wrong marginals delivers large positive biases, instead. If the dependence structure is represented by a copula which is not elliptical, e.g. the Clayton copula, the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of marginal skewness. Extensive Monte Carlo studies then show that the misspecifications in the marginal volatility equation more than offset the biases in copula parameters when VaR forecasting is of concern, small samples are considered and the data are leptokurtic. The biases in the volatility parameters are much smaller, whereas those ones in the copula parameters remain almost unchanged or even increase when the sample dimension increases. In this case, copula misspecifications do play a role for VaR estimation. However, these effects depend heavily on the sign of the dependence.

1. Introduction

Value at Risk (VaR) has become one of the most popular risk measures since it was recommended and adopted by the Bank of International Settlements and USA regulatory agencies in 1988. Jorion (2000) provides an introduction to Value at Risk as well as discussing its estimation, while the www.gloriamundi.org website comprehensively cites the Value at Risk literature as well as providing other VaR resources. Recently, Ané and Kharoubi (2003), Junker and May (2005) and Fantazzini (in press) investigated the influence of different bivariate joint distribution assumptions on the Value at Risk. They found that the best results can be achieved by models allowing for heavily tailed marginals and a modelling of tail dependency and asymmetric tail dependency. However, they also found out that, for short positions, a multivariate normal model with dynamic normal marginals and constant normal copula can be a proper choice.

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The aim of this paper is to investigate how misspecifications both in the marginals and in the copulas can affect the estimation of the Value at Risk when dealing with multivariate portfolios. Given the partitioning of the multivariate parameter vector into separate parameters for each margin and parameters for the copula, it has become usual practice to break up the optimization problem into several small optimizations, each with fewer parameters. This multi-step procedure is known as the method of Inference Functions for Margins (IFM); e.g., see Patton (2004), McNeil et al. (2005), Patton (2006a), Cherubini et al. (2004), Gunky et al. (2007) and Karlis and Nikoloulopoulos (2008). Therefore, in order to achieve our aim, we first analyze the effects of misspecified marginals on the multivariate parameter vector estimation, while we examine in a second step the effects of model misspecification on the computation of the Value at Risk. Regarding the first point, we find the interesting result that, when there is skewness in the data and symmetric marginals are used, the estimated normal copula correlations are negatively biased, and the bias increases when moving from the Student's t to the normal distribution, reaching values as high as 27% of the true values. We find that the bias almost doubles if negative correlations are considered, compared to positive correlations. When the true dependence function is represented by the t copula, the choice of the marginals tends to have much stronger effects on copula parameter estimation, with biases up to 50% of the true values for the correlations and up to 380% for the t copula degrees of freedom parameter. If the dependence structure is represented by a copula which is not elliptical, e.g. the Clayton copula, the effects of marginal misspecifications on the copula parameter estimation can be rather different, depending on the sign of the marginal skewness.

The second contribution of the paper is an extensive Monte Carlo study carried out to assess the potential impact of both misspecified margins and misspecified copulas on the estimation of multivariate VaR for equally weighted portfolios. We find that, when small samples are considered and the data are leptokurtic, the overestimation/underestimation in the marginal volatility parameters is so large as to offset the negative biases in copula parameters, thus delivering very conservative VaR estimates for all quantiles. This is true for all multivariate models considered in the analysis.

When the sample dimension increases, the biases in the volatility parameters are much smaller, whereas those in the copula parameters remain almost unchanged or even increase, like for the t copula and the Clayton copula parameters, when (wrong) symmetric marginals are used instead of skewed ones. In this case, copula misspecifications do play a role. However, these effects depend heavily on the sign of the dependence: if it is negative, the bias can be as large as 70%, like for t copula correlations, for instance. If it is positive, the bias is much smaller (10% or less for the t copula correlations), and the effects on quantile estimation are much more limited, if not completely offset by marginal misspecifications. Therefore, this Monte Carlo evidence gives some insights into why previous empirical literature found that the influence of a misspecification in the copula is given with 20% or less of the whole estimation error for the VaR; see e.g. Ané and Kharoubi (2003) and Junker and May (2005).

Finally, we perform an empirical analysis with ten trivariate portfolios, where we quantify the risk of the portfolio under different joint distribution assumptions. The rest of the paper is organized as follows. Section 2 presents the copula–VAR–GARCH models, while in Section 3 we perform simulation studies to assess the finite sample properties of these models under different DGPs. In Section 4, we conduct an empirical analysis with ten portfolios composed of stocks quoted at the NYSE, while we conclude in Section 5.

2. Copula–VAR–GARCH modelling

Consider a general copula–vector autoregression model, where the n endogenous variables $x_{i,t}$ are explained by an intercept μ_i, autoregressive terms of order p, and an error term $\sqrt{h_{i,t}}\eta_{i,t}$:

$$
\begin{aligned}
x_{1,t} &= \mu_1 + \sum_{i=1,}^{n} \sum_{l=1}^{p} \phi_{1,i,l} x_{i,t-l} + \sqrt{h_{1,t}}\, \eta_{1,t} \\
&\vdots \\
x_{n,t} &= \mu_n + \sum_{i=1,}^{n} \sum_{l=1}^{p} \phi_{n,i,l} x_{i,t-l} + \sqrt{h_{n,t}} \eta_{n,t}.
\end{aligned}
\tag{1}
$$