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A genetic algorithm estimation of the term structure of interest rates

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ABSTRACT

The term structure of interest rates is a key instrument for financial research. It provides relevant information for pricing deterministic financial cash flows, it measures economic market expectations and it is extremely useful when assessing the effectiveness of monetary policy decisions. However, it is not directly observable and needs to be estimated by smoothing asset pricing data through statistical techniques. The most popular techniques adjust parsimonious functional forms based on bond yields to maturity. Unfortunately, these functions, which need to be optimised, are highly non-linear which make them very sensitive to the initial conditions. In this context, this paper proposes the use of genetic algorithms to find the values for the initial conditions and to reduce the risk of false convergence, showing that stable parameters are obtained without imposing arbitrary restrictions.

1. Introduction

The term structure of interest rates is defined as the relationship between the nominal interest rates of default-free zero-coupon bonds and time to maturity. It is well known that the term structure of interest rates is an extremely useful tool, not only for finance, but also for macroeconomics. In particular, it may be employed to price financial securities, to risk management and to portfolio and corporate financial decisions. It is also a key element for the implementation and evaluation of monetary policy.

The non-arbitrage hypothesis of the term structure implies that the functional relationship must have two basic characteristics: continuity and smoothness. The latter guarantees the continuity of the implicit term structures of forward interest rates, which is also required in a general non-arbitrage financial context. Nevertheless, the absence of government zero-coupon bonds for all maturities implies that, in practice, the actual term structure is unobservable. Hence, the researcher must estimate the term structure using, on the one hand, the limited number of spot rates directly observed from the prices of zero-coupon government bonds and, on the other, the prices of government coupon bonds. Given that, from both a theoretical and a practical point of view, coupon bonds can be understood as portfolios of zero-coupon bonds, the researcher has in fact enough data to cover the entire time span of the term structure of interest rates.

These difficulties have given rise to a vast line of research focussed on obtaining the best representation of the underlying term structure on the basis of market prices of government bonds. From the beginning of this research by Guthmann (1929) until now, we have gone from the use of graphic, subjective and handmade methods (Anderson et al., 1996) to the use of sophisticated non-parametric methods based on gradually smoothed spline functions fitting (Anderson and Sleath, 2001).

In this context, simple functional forms that describe the whole time span of the structure of interest rates, and that are parsimonious in parameters and easy to implement, have been and still are the tools most widely used by central banks.

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To be more precise, the function developed by Nelson and Siegel (1987) and its augmented version, proposed by Svensson (1994) (hereafter the NSS functions), are currently used by nine of the thirteen central banks reporting their estimations to the Bank for International Settlements (BIS, 2005). The financial literature contains other proposals of simple functional forms, although they are less used in practice. Some examples are McCulloch (1971), Chambers et al. (1984), Diament (1993) and Mansi and Phillips (2001).

The choice of these simpler techniques is fully justified when their use is restricted to the analysis of investments and monetary policy. Other uses of the term structure of interest rates, such as those related to non-arbitrage valuation and the pricing of fixed-income assets and derivatives, probably need more accurate estimates of the term structure. In any case, the empirical evidence reported by Dillén and Pettersson (2005) suggests that "the quantitative difference between the augmented Nelson and Siegel method and the smoothed splines approach is normally very small". Moreover, there are well known examples in which the estimated parameters of these simple polynomial functions have been used in financial pricing techniques for modelling government bond yields, either directly (Diebold and Li, 2006; Diebold et al., 2006), or adding macroeconomic factors (Diebold et al., 2006; Diebold et al. 2005).

One of the most important technical questions in the day-to-day fitting process of the NSS functions is, undoubtedly, the possibility that the parameters obtained may correspond to a local optimum and not to the global optimum, i.e. the risk of false convergence. This risk comes from the high non-linearity of the functions to be fitted, and in practice it is often found after the estimation due to the poor fit relative to conventional levels.

This problem also rises from the high sensitivity of the estimated parameters to the initial conditions used in implementation of the usual non-linear optimisation algorithms, i.e. maximum likelihood and non-linear least squares. The main empirical consequence of these convergence problems is that, once the initial conditions are given, the estimated parameters exhibit great variability regardless of the procedure followed for their determination.

The objective of this paper is to develop an estimation procedure for these functions that avoids the problem caused by the choice of the initial values in the optimisation process, which, of course, always entails some degree of discretion. To this end, we propose the use of genetic algorithms in the process of fitting the NSS functions to the term structure of interest rates. The results are quite satisfactory because, besides resolving the risk of false convergence, the use of genetic algorithms generates series of parameters with less volatility and a more accurate fit to the underlying term structure of interest rates.

Genetic Algorithms (GAs) have been successfully used in the parameter estimation of non-linear models such as some stochastic ordinary differential equations (Alcock and Burrage, 2004), or when some threshold is added to the model, like in Baragona et al. (2004) or Yang et al. (2007). More importantly, Fernández-Rodríguez (2006) applies a GA to the term structure modelling using free-knot splines.

The rest of this paper is structured as follows: Section 2 describes the NSS functions; Section 3 poses and analyses the estimation of the term structure in the Spanish Government bond market using traditional optimisation methods; Section 4 reports the results of re-estimating the term structure using genetic algorithms. Moreover, we compare them with the results obtained under traditional non-linear least squares optimisation. Finally, Section 5 concludes.

2. The Nelson–Siegel–Svensson functions

Nelson and Siegel (1987) propose a parametric model, in which the instantaneous forward rates at time t on an investment initiated m periods in the future and which matures some given period beyond the start date of the contract, have the following exponential expansion functional form,

$$f_t(m) = \beta_{t,0} + \beta_{t,1} \exp\left(\frac{-m}{\tau_t}\right) + \beta_{t,2} \frac{m}{\tau_t} \exp\left(\frac{-m}{\tau_t}\right), \quad (1)$$

where $\beta_{t,0}$, $\beta_{t,1}$, $\beta_{t,2}$ and τ_t are the parameters to be estimated.

This function, $f(m)$, has convenient and desirable characteristics for capturing the shape of the term structure. One key characteristic is the existence of the limit of the function $f(m)$ for $m \to \infty$ and for $m \to 0$, i.e.:

$$\lim_{m\to\infty} f(m) = \beta_{t,0}$$

and

$$\lim_{m\to 0} f(m) = \beta_{t,0} + \beta_{t,1}, \quad (2)$$

which allows the computation of both very long and very short-term instantaneous forward interest rates.

The way in which the transition occurs between the very short-term rate and the very long-term rate is captured by parameters $\beta_{t,2}$ and τ_t. The curve would have a "U" shape, an inverted "U" shape, or a sine-shaped function rather than one with a single local optimum depending on whether $\beta_{t,2}$ is negative, positive or close to zero respectively. The speed at which the forward interest rate approaches its very long-term value is given by τ_t^{-1}.

Given the forward curve, we can determine the spot rate on a zero-coupon bond with m periods to maturity, denoted by $s_t(m)$, by taking the equally-weighted average over the forward rates:

$$s_t(m) = \frac{1}{m} \int_0^m f_t(u)\mathrm{d}(u). \quad (3)$$